Multi-mode Vortex-Induced Vibrations of a Flexible Circular Cylinder

by

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Vortex-Induced Vibration (VIV) is a key issue in deep water riser design. Water depths up to 3000 m are found in typical oil extraction areas and in order to extract the oil from the sea bed, flexible slender pipes called risers, are necessary. Risers can experience VIV when exposed to marine currents, because the shedding of vortices downstream of the structure induces forces on it, which may cause vibrations. Design of deep water systems is a challenging engineering problem and the understanding of VIV and its suppression are very active areas of research. The study of this problem is hampered by a lack of good quality measurements and a lack of totally reliable models to predict the response and the fluid loading on risers undergoing VIV.

A vertical tension riser model in a stepped current was used to study its response when subject to VIV. The riser, 28 mm in diameter, 13.12 m long and with a mass ratio (mass/displaced mass) of 3.0, was tested in conditions in which the lower 45% was exposed to a uniform current at speeds up to 1 m/s, while the upper part was in still water. Reynolds numbers reached values of approximately 28000. The response in the in-line and cross-flow directions was inferred from measurements of bending strains at 32 equally spaced points along its length. Cross-flow vibrations were observed at modes up to the $8^{th}$ and in-line up to the $14^{th}$. The response included significant contributions from several modes, all at a frequency controlled by lock-in of the dominant mode. Measurements of in-line and cross-flow displacements reveal a strong dependence on the motion. A finite element method technique has been developed to model the riser and used in conjunction with the experimental response data, to calculate in-line and cross-flow force distributions along the axis of the riser.
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En memoria de mi padre.
Publications

Journal papers


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Nomenclature

Roman symbols

\( c_{x,y} \)  Curvature in the \( x \) or \( y \) direction
\( f \quad \text{External force} \)
\( f_i \quad \text{Inertial force} \)
\( f_1 \quad \text{Fundamental natural frequency} \)
\( f_{dr,y} \quad \text{In-line or cross-flow dominant frequency} \)
\( f_s \quad \text{Vortex shedding frequency} \)
\( f_z \quad \text{Axial external force} \)
\( h^e \quad \text{Generic finite element height} \)
\( \ell \quad \text{Lagrange interpolator - Shape function} \)
\( m \quad \text{Mass per unit length} \)
\( m^* \quad \text{Mass ratio} \)
\( m^*_b \quad \text{Bumpy cylinder’s mass ratio} \)
\( n^e \quad \text{Number of finite elements} \)
\( \mathbf{r} \quad \text{Displacements vector} \)
\( u \quad \text{In-line displacements} \)
\( v \quad \text{Cross-flow displacements} \)
\( w \quad \text{Axial displacements} \)
\( w_s \quad \text{Submerged weight} \)
Nomenclature

\( A \)  
Riser cross-sectional area

\( A_{x,y} \)  
\( x \) or \( y \) direction displacement modal amplitudes matrix

\( A_{c_{x,y}} \)  
\( x \) or \( y \) direction curvature modal amplitudes matrix

\( B \)  
Connectivity matrix

\( C \)  
Proportional damping matrix

\( C_{x,y} \)  
In-line or cross-flow curvatures matrix

\( D \)  
External diameter

\( E \)  
Elasticity modulus

\( EA \)  
Axial stiffness

\( EI \)  
Flexural stiffness

\( F \)  
Reduced nodal forces vector

\( I \)  
Cross-sectional inertia

\( K \)  
Stiffness matrix

\( KC \)  
Keulegan-Carpenter number

\( L \)  
Total length

\( L_s \)  
Submerged length

\( L \)  
Lagrange interpolators matrix - Shape functions

\( M \)  
Moment

\( M \)  
Consistent mass matrix

\( N_a \)  
Number of degrees of freedom in the axial finite elements

\( N_t \)  
Number of degrees of freedom in the transverse finite elements

\( P^e_{x,y,z} \)  
Elemental mapping matrices for the transverse and axial cases

\( Q \)  
Secondary variables vector

\( Re \)  
Reynolds number

\( S \)  
Strouhal number

\( T \)  
Tension

\( T_t \)  
Top tension

\( T_b \)  
Bottom tension

\( U \)  
In-line displacements matrix

\( V \)  
Cross-flow displacements matrix
Nomenclature

- $V$: Free stream flow speed
- $V_i$: Reduced velocity based on the fundamental natural frequency in still water
- $V_{dx,dy}$: Reduced velocity based on the response dominant frequencies
- $\mathbf{W}$: Axial displacements matrix
- $\tilde{s}$: Root mean square value of $s$
- $\hat{s}$: Maximum value of $s$
- $\check{s}$: Minimum value of $s$
- $\bar{s}$: Mean value of $s$
- $e$: Superscript refers to a generic finite element
  - $\underline{\text{underline}}$: Refers to the global finite element system

Greek symbols

- $\alpha, \beta$: Proportional damping coefficients
- $\epsilon$: Axial strain
- $\Phi$: Displacement mode shapes matrix
- $\Phi_c$: Curvature mode shapes matrix
- $\rho$: Water density
- $\sigma$: Axial stress
- $\tau$: Shear force
- $\Psi$: Hermite interpolators matrix - Shape functions
- $\psi$: Hermite interpolator - Shape function
- $\vartheta$: Weight function
- $\xi_i$: $i$th mode damping ratio
- $\mu$: Dynamic viscosity
Abbreviations

2D  Two-dimensional
3D  Three-dimensional
CFD Computational Fluid Dynamics
DNS Direct Numerical Simulation
dof degree-of-freedom
FEM Finite Element Method
PIV Particle Image Velocimetry
RMS Root Mean Square
SCR Steel Catenary Risers
TLP Tension Leg Platform
VIV Vortex-Induced Vibrations
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Chapter 1

Introduction

Vortex-induced vibration (VIV) is a multidisciplinary field which involves fluid mechanics, structural mechanics and vibrations, complex techniques for data analysis and numerical methods applied to fluid dynamics and solid mechanics. Many engineering problems encounter VIV in their designs. Engineering applications such as power transmission lines, industrial chimneys and stacks, tube arrays in heat exchangers, bridge decks and high rise buildings are some examples. The offshore industry has been at the forefront of the interest in this fluid-structure interaction subject, in the past decades. Drilling and production risers, marine cables, mooring lines, seabed pipelines and towed cables are just some examples of marine engineering applications where VIV plays an important role. In the past 40 years the water depths of the new explorations projects have increased from around 100m in 1965 to about 3000m in 2003, (Richardson et al. 2004). Deep water flexible structures are prone to experience VIV and hence they must be carefully designed in order to avoid environmental and economical catastrophes.

Suppression, reduction, prediction and control of VIV (specially without increasing the drag) is the ultimate objective and requires the understanding of the physics involved in VIV. Experimentally, laboratory tests are the best way of producing high quality data because all the input parameters to the system are under control. In field experiments, the first problem is that not even the current may be known to the required degree
of certainty. On the other hand, in a laboratory it is very difficult to achieve realistic fluid and structural parameters (Reynolds numbers, aspect ratios, mass ratios, etc...).

In the past years with the increase of the computer power, numerical techniques have increased the amount of data obtained and more benchmarks are now possible. Although better computers are becoming available to researchers gradually, they are still extremely expensive to use due to the time consumption. Computational techniques are still not available to be used in design stages and more work needs to be done. Empirical codes are those based on experimental data, they are easy and fast to use, that is why they are broadly used.

The second chapter of this thesis contains a brief summary of the literature regarding flow around stationary circular cylinders and vortex-induced vibrations on rigid cylinders flexibly mounted with one or two degrees of freedom. The last section of that chapter focuses on reviewing literature regarding VIV on flexible cylinders. Chapter 3 describes the mathematical model used to describe the riser model used for the experiments, and the numerical technique, the finite element method (FEM), used to approximate the solution of the governing differential equations. In the fourth chapter the experimental set-up used is described. Chapter 5 presents a detailed description of the data analysis process and the results produced with these methodologies. In the sixth chapter one can read how the hydrodynamic forces acting on the riser have been calculated, by using the mathematical model presented in chapter 3 and the experimental data obtained from the experiments. In the last part of the thesis, a discussion about the results and the conclusions are presented together with suggested further work.
Chapter 2

Literature review

In this chapter a review of previous work on the subject is presented. The emphasis is given to the last section of this chapter, research related to high mode number VIV of flexible circular cylinders. A brief introduction is devoted to the flow around stationary cylinders and to VIV of flexibly mounted rigid cylinders with one and two degrees of freedom. It is outside the scope of this thesis to provide an extensive review of these topics and the author intends to give a general short introduction. For more details, the reader can follow the many references cited in the text.

2.1 Flow around stationary circular cylinders

The flow around a a bluff body such as a circular cylinder is characterized by large zones of separation. An adverse pressure gradient in the direction of the flow, imposed by the shape, causes the boundary layer to separate from the surface of the cylinder generating a recirculation in the flow. The wake of a cylinder is formed as a consequence of this phenomena and it has been studied extensively in the past centuries. Leonardo Da Vinci in 1497 was probably, the first to pay attention to how the flow separates from the surface of different objects and what types of patterns were formed in their wakes. Many other researchers since then have been studying the flow around different
objects. In this section there is a brief summary, of how the wake of a circular cylinder evolves as a function of different parameters. An excellent and extensive description of the flow around a cylinder can be found in Zdravkovich (1997).

Reynolds (1883) discovered while investigating internal flows in pipes that the transition between laminar and turbulent states was a function of the density, the viscosity, the velocity of the fluid and of the diameter of the pipe. The Reynolds number gives a relation between inertial forces and viscous forces in the flow, and is defined as:

$$Re = \frac{\rho V D}{\mu} \quad (2.1)$$

with $\rho$ being the fluid density, $V$ the flow speed, $D$ the cylinder diameter and $\mu$ the fluid viscosity. He determined that the transition to turbulence happened within a range of Reynolds numbers. One can observe the phenomenon of separation and the transition to turbulence, by increasing the Reynolds number from very low regimes.

The flow regimes around a stationary circular cylinder are governed by the Reynolds number and so, transition states can be described by the $Re$ and the modifications that occurs due to other influencing parameters. Bloor (1964) found transition in the shear layers, as the $Re$ was rising, the transition zone moved towards the separation points changing the length and the width of the near wake region. A transition in the boundary layers advancing towards the stagnation point was also shown, resulting in turbulence in all the zones of disturbed flow. Table 2.1 shows the states of the flow as compiled by Zdravkovich (1997).

In the non-separation regime the flow remains attached to the surface of the cylinder and the steady laminar wake is formed. With a $Re$ of 4, separation begins and a closed near wake is formed with a weak recirculation consisting of two symmetric eddies (Föppl’s twin vortices). The shear layers meet at a point named the confluence, neutral or free stagnation point, closing the near wake region. When the Reynolds number reaches 47 an instability in the near wake is observed, leading to an oscillation of the shear layers at the neutral point. There is a common feature for circular cylinder
flows with Reynolds numbers greater than 47, that is vortex shedding and the formation of the Kármán-Bénard vortex street. Vortex shedding is the result of an interaction between the shear layers in the near body wake involving the vorticity inherent to them. Flow instability, as mentioned by Gerrard (1966), produces vortices that grow alternately on each side of the body. The shedding of a vortex is produced when the shear layer of the next upcoming vortex interacts with the opposite side shear layer, cancelling its vorticity because of its different sign. This process is repeated at each side of the cylinder, generating the vortex street in the wake, figure 2.1.

The frequency of vortex shedding, that appears about a $Re$ of 47, can be expressed non-dimensionally as the Strouhal number:

$$S = \frac{f_s V}{D} \tag{2.2}$$

where $f_s$ is the shedding frequency. Experiments from different researchers have shown
that the frequency of vortex shedding in the wake of a cylinder is a function of the
Reynolds number, the roughness of the cylinder surface, the turbulence intensity in the
incoming flow, the aspect ratio, the blockage ratio, etc. Figure 2.2 shows the variation
of $S$ with $Re$. The product between the shedding frequency and the flow speed divided
by the diameter of the cylinder is practically constant in the subcritical regime, with
a value of approximately 0.2.
Reynolds number is the governing parameter for the flow around a stationary circular cylinder, but other influencing parameters can become governing over a certain value. Turbulence intensity, surface roughness, aspect ratio and oscillations in flexibly mounted cylinders are important influencing parameters that can affect the transition in the different disturbed regions. End effects and wall blockage play an important role when using test models.

### 2.2 Important parameters for the analysis of vortex shedding

- **Reynolds number** (Re) as defined in equation 2.1, is an important parameter to characterize the flow.

- **Turbulence intensity** Is a measure relative to the free stream flow velocity that indicates the amount of disorder of the flow. Usually it is expressed as the ratio between the rms value of the velocity and the free stream velocity.

- **Aspect ratio** $L/D$, is the relation between the length and the diameter of the cylinder.

- **Roughness ratio** Calculated as $k/D$ where $k$ is the equivalent sand roughness parameter. It measures the importance of the surface roughness to the diameter of the structure.

### 2.3 Important parameters for the analysis of VIV

- **Damping ratio** ($\zeta$) It is an expression for the ability of the structure to dissipate energy during a cycle of vibration. We can find experimental damping factors by carrying out decay tests. See section 5.3 of this thesis for more details.

- **Reduced velocity** ($V_r$)
In this thesis if the fundamental natural frequency of the system in air \( f_1 \) is used, it will be referred as \( V_1 \). If the dominant vibrating frequency \( f_{dx,y} \) is used the reduced velocity will be referred as \( V_{dx,y} \).

\[ V_r = \frac{VT}{d} = \frac{V}{fD} \quad (2.3) \]

- **mass ratio** \( (m^*) \) Ratio between the mass per unit length \( (m) \) of the structure and the displaced mass of fluid when oscillating.

\[ m^* = \frac{m}{\rho \frac{\pi}{4} D^2} \quad (2.4) \]

### 2.4 Vortex-Induced Vibrations (VIV)

#### 2.4.1 VIV of rigid cylinders: One degree of freedom

#### 2.4.1.1 Transverse motion

Vortex shedding in the wake of a stationary circular cylinder occurs for Reynolds numbers greater than 47. Under this situation, periodic shedding is formed downstream of the cylinder, creating a situation in which alternating fluid forces are generated on the cylinder, and if it is flexibly mounted and it has suitably low damping and reduced mass, vortex-induced vibrations (VIV) can occur. Once the oscillations start, the flow is modified and the excitation affecting the moving cylinder changes which again modifies the response and so on. This feedback process between the structure and the flow, leads to a very complex non-linear interaction. The mechanisms that are present in this interaction are not completely understood, that is why predictions of how the flow affects the cylinder producing the response and vice versa, are not completely reliable. Excellent review works on VIV have been published by Sarpkaya (1979) and (2004), Bearman (1984), Parkinson (1974) and Williamson & Govardham (2004).
A classic experiment reviewing the behaviour of a flexibly mounted cylinder restricted to move transverse (cross-flow) to the flow was carried out by Feng in (1968). Feng’s experiment shows many important details of Vortex-Induced Vibrations and that is why his work is often referenced. Feng analyzed a system with high mass ratio \( (m^* \simeq 250) \), using air, by measuring the amplitude response, the oscillation and shedding frequencies and surface pressures. He calculated the phase angle between the oscillating force and the response by analyzing the displacements and the surface pressures. The data is shown in figure 2.3 as appeared in Parkinson (1989). Feng increased the speed of the flow in small steps to observe the response of the circular cylinder due to the current. No vibration happened until a reduced velocity \( V_1 = 4 \), at this point vibration appeared and continued up to a reduced velocity of approximately \( V_1 = 8.5 \). In the response range, two separate branches could be observed, one achieving higher amplitudes than the other. These two branches have later been called initial and lower branch, and it was observed that the jump in displacement between the two branches, was associated with an abrupt change in phase angle. The frequency of vibration was very close to the natural frequency of the system in air, in the whole range where non-zero response was found. Vortex shedding frequency followed the Strouhal relationship for a stationary cylinder, except in the range of reduced velocity that goes from 5 to 7, where the frequency of oscillations and the frequency of vortex shedding were equal, and near the natural frequency of the system. This phenomena, where oscillation frequency and vortex shedding frequency were roughly equal, was called lock-in, and it was where the higher amplitude responses were observed. The lock-in is also known as resonance, synchronization or wake capture. In the lock-in range, vortex shedding is not controlled by the Strouhal law but by the cylinder motion. At the upper end of the lock-in range a sudden jump changes the frequency of vortex shedding again, to the value predicted by the Strouhal relationship for a stationary cylinder. It was also noticed that there was a hysteretic behaviour, the amplitudes were higher when increasing the flow speed than when decreasing it. This infers that the system has ”memory” and responds differently to different flow time histories.

Anand & Torum (1985) experiments carried out in water with low mass ratio, can
be compared to Feng’s. The case of low mass ratio introduces some differences in the frequencies observed. These differences occurred because of the effect of the added mass. The main feature is that for the case of low mass ratio, the frequency of vibration in the lock-in range is not as close to the natural frequency of the system as it is in air. It increases linearly with $V_1$ inside the synchronization range. Another important difference is that the lock-in range extends over a larger span of reduced velocity. Figure 2.4 shows Anand’s response and frequency data as presented in Sumer & Fredsoe (1997). Recent studies by several authors exploring low mass and damping cylinders, Khalak & Williamson (1996) and Brankovic (2004), have confirmed how bodies can oscillate with large amplitudes even though the oscillation frequency is not close to the natural frequency of the body, changing the classic definition of lock-in to a newer one, adopted.
by Sarpkaya (1995), in which the frequency of vortex shedding has to be equal to that of the oscillation. Under this situation, the excitation lift force is in phase with the motion, and it fluctuates at the same rate as the structure.

Figure 2.4: Transverse response of a freely vibrating cylinder in air, from Anand’s experiment, as appears in Sumer & Fredsøe (1997)

Williamson & Govardham (2004) continuing the research started by Williamson & Roshko (1988), identified the different vortex structures in the wake of an oscillating cylinder and they associated different patterns to each response branch found in amplitude vs. reduced velocity diagrams. What they named the 2S (2 single vortices per cycle) was initially found in free vibration of the cylinder. A second pattern called 2P (2 pairs of vortices per cycle) appeared in free and forced vibrations and a third pattern, the P+S (1 single + 1 pair of vortices per cycle), appeared initially only in forced vibrations. They defined a vortex mode map, figure 2.5, in which the vortex structures in the wake of a circular cylinder appear as a function of the reduced velocity and the response amplitude.
They compared the response of high mass ratio systems, using Feng’s data, against their low mass-damping data. The reduced damping ($S_G$, see equation 5.33 in section 5.4.1), of their data was 3% of that corresponding to Feng’s. Figure 2.6, shows how in the low mass ratio system, the synchronization range was 4 times larger, and the oscillation frequency did not match the natural frequency, it was around 1.4 times bigger in the lower branch (for a $m^* = 2.4$). They observed an intermediate branch with larger amplitudes between the initial and lower branches, the upper branch. There was a hysteretic behaviour between the initial and the upper branch, but between the upper and the lower branch there was an intermittent change. The 2S mode was found in the initial branch and the 2P in the upper and lower branches. The jumps between the three branches were associated with changes in the phase angle between the motion and the excitation force. Govardhan & Williamson (2002) suggested the existence of a limiting value of mass ratio, depending on the shape of the structure, under which lock-in would persist for any given reduced velocity, and de-synchronisation would never happen independently of how much the reduced velocity was increased. They suggested a value of $m = 0.54$ for a circular cylinder.
The maximum amplitude achievable by a certain system is something that has captured the interest of researchers working on VIV. Griffin (1982) published a work where the relationship between the maximum amplitude recorded for a system was plotted against a parameter called reduced damping, which was based on the mass and the damping ratio of the system. They stated that as the reduced damping ($S_G$) increased the amplitude of the response decreased until motion disappeared. An extensive compilation of non-dimensional maximum amplitude response data from several authors is presented in Williamson & Govardham (2004). In figure 2.7 data obtained from cylinders in air and water is presented.

Figure 2.6: Comparison of low (Williamson) and high (Feng) mass ratio data from Williamson & Govardham (2004)
2.4.1.2 In-line motion

If a cylinder is flexibly mounted in the in-line direction, the oscillating drag produced by vortex shedding may induce in-line vibrations. In general this vibration will occur at roughly twice the frequency for cross-flow vibration. In more detail, we can distinguish two response regions in the reduced velocity domain, in the range of reduced velocities from around 1.5 to 3.5 (King et al. 1973).

- **First instability region** \( (V_1 < \frac{1}{2S}) \) The vibrations in this region are caused by the symmetric shedding of vortices produced by the oscillation of the cylinder (King et al. 1973). The force generated by the vortices is in phase with the body velocity. This type of response appears for reduced velocities less than \( 1/2S \) as explained by Bearman (1984) from Wooton et al. (1974). This first instability region has been observed in water but not in air.

- **Second instability region** \( (V_1 \simeq \frac{1}{2S}) \) Another response region is found at reduced velocities of approximately \( \frac{1}{2S} \). In this region flow visualization by King et al. (1973) showed symmetric shedding. As the in-line force frequency gets closer to the natural frequency, the amplitude of vibrations increase in a range...
called the second instability region. The amplitudes registered in this region are larger than in the first instability region but still small compared to the cross-flow oscillations.

2.4.2 VIV of rigid cylinders: two degrees of freedom

Studies have been extensively carried out on one-degree-of-freedom (1dof) cylinders, restricted to transverse motion, and research with rigid cylinders allowed to move in both in-line and transverse to motion, is not common. There is a debate about if results obtained with 1 dof cylinders, free to move in either the in-line of the transverse directions, can be assumed to be similar to those obtained with 2 dof systems. Jauvtis & Williamson (2003) set up an experiment in which the natural frequency of the cylinder in the in-line and transverse directions was the same. They found that the freedom to oscillate simultaneously in-line and transverse to the flow practically does not modify the response branches, the forces and the vortex wake modes, in cylinders with mass ratios greater than 6. For bodies with mass ratios lower than 6, they found a new response branch with peak amplitudes considerably large (around 1.5), and with a vortex structure in the wake formed of two triplets of vortices per cycle (2T).

Jeon & Gharib (2001) reported experiments with a circular cylinder oscillating with 1 and 2 dof in a low speed water tunnel. They used Digital Particle Image Velocimetry (DPIV) to identify the vortex structures in the wake and strain measurements to measure the forces. They found several qualitative differences when the in-line dof was allowed. The disappearance of the 2P mode of shedding, observed in transverse motion studies, and the fact that the in-line motion was affecting the phase between the lift and the transverse motion, were the main differences observed. The values of circulation were found to be higher in the 2 dof case.
2.4.3 VIV of flexible circular cylinders

Vortex Induced Vibrations affecting flexible structures is a classic engineering problem. Since the 60’s the offshore industry has been involved in research with the aim of improving the design of offshore structures and risers. But it was in fact during the 80’s when the high current profiles found in deep waters, such as the Gulf of Mexico or the Brazilian coast, with water depths up to 3000 m, increased the need for a better understanding of the VIV phenomena. Figure 2.8 shows how in the last 40 years the exploration depths has increased from 100 m to more than 3000 m.

![Figure 2.8: Evolution of the water depth in new explorations, Gulf of Mexico, from Richardson et al. (2004)](image)

Prediction of VIV is an important aim, but the ultimate aim of the research efforts of the last decades is to be able to control and suppress it. One of the issues that arises is that there is not the same amount of quality data regarding flexible cylinders, as there is for the case of rigid cylinders, and results in most of the cases are not directly applicable from one physical situation to another.

Traditionally, the key issues when designing realistic test campaigns for long flexible
models are:

− The number of sensors has to be adequate to resolve the modal shapes. Sometimes cost and physical installation of the instrumentation limits the number of sensors to install and hence the number of modes to resolve. The situation of equally spaced sensors along the length in a number equal to the highest mode expected is the ideal situation which cannot always can be achieved.

− It is very difficult to reach realistic Reynolds numbers in laboratory experiments. The size and technical characteristics of the facilities are limiting factors. Flow visualization techniques and other quantitative flow characterization tools are very difficult to use in large scale facilities.

− Accelerometers may be affected by gravity when risers are tilted, introducing non-desired signal amplitudes. Corrections need to be done in the data analysis process, so the selection of the right type of instrumentation is crucial.

− The evolution of the top and bottom end tensions is very important, and together with the right displacement instrumentation can lead to force data, not common in the literature.

− There is a need to measure forces on the cylinders at different points along the length, without interfering with the flow. There is no available data on this aspect in past literature to the knowledge of the author.

− The supporting and auxiliary structures used to hold the models need to be carefully designed in order to have natural frequencies far away from the expected excited ones during the experiments.

The fact that vibrations of flexible structures are characterized by multi-modal and multi-frequency vibrations adds complexity to the cylinder VIV problem. The flexural stiffness of the material and the axial tension are very important parameters to take into account regarding the overall response. Under normal operation, vortex shedding would excite several modes of vibration in the structure. If uniform flows are affecting
the structure, one should observe only one dominant frequency in the response, coming from the only excitation source, the vortex shedding. Many experiments have been performed in the past, with full scale operating facilities and in laboratories under sheared currents, but reviewing the recent literature the mechanisms that govern the vortex-induced vibrations are still not clear, and prediction tools are not reliable. Here are several basic questions researchers have been formulating in the past years when working with flexible structures, and some of the questions this thesis tries to answer:

- Which modes of vibration are going to be excited under specific current conditions?
- What amplitudes are we going to observe?
- How are these maximum amplitudes distributed along the riser?
- What is the relationship between in-line and cross-flow displacements?
- What is the contribution of each mode to the global response of the structure?
- What modes and frequencies will be dominant under specific conditions?
- Will we observe several modes running at the same frequency?
- Are there traveling waves in the riser, and what are their effects on the overall response?
- What is the effect of the varying tension on the natural frequencies and on the overall amplitude response?
- How are the forces generating the vibration?

Vandiver (1998) comments that the challenges are still the lack of understanding of the fluid-structure interaction physics, the need for adequate structural models, the need for high quality multi-mode full scale data, the necessity to further develop data analysis techniques to better understand the results and the industrial need for effective suppression devices. Whilst the structural modeling has positively evolved together
with the data analysis techniques in the past decade, the mechanisms that rule the fluid-structure interaction and the need for full scale or laboratory data, are still a major concern.

In this section an overview of past work carried out on flexible cylinders is presented. A review of the most relevant experimental work exclusively focused on flexible circular cylinders is presented with the aim of giving the reader a general view of what is nowadays the state of the experimental research in this field. Following, a discussion of the state of the art, the results produced by different empirical and CFD based prediction methods, is presented. It is not the intention of the author to describe the codes, but just to comment on the output derived from their use.

2.4.3.1 Experiments

Vandiver (1983) reports experiments on a long flexible cylinder subjected to a steady tidal uniform flow at Castine, Maine. He used two 22.86 m length long circular cylinders, one with an external diameter of 32 mm and another one with 41 mm giving and aspect ratios of around 750 with Reynolds numbers up to 10000. The data analysis was focused on drag coefficient calculations. Under lock-in conditions, drag coefficients in excess of three were observed, with single mode/single frequency lock-in response behaviour. A ratio of 2 between the in-line and the cross-flow frequency was observed, with considerably higher cross-flow motions with respect to those in-line. Non lock-in conditions were also observed in which the overall response was composed of the contribution of up to 4 in-line and cross-flow modes. The frequency bandwidth of the local drag and lift changed from a single frequency at lock-in to a band limited random process. Anti-node amplitudes of up to 1 diameter were observed under lock-in conditions. The response was found to be insensitive to variations in Reynolds number and to large amounts of surface roughness (helical lays of rope). He proposed an expression to predict the drag coefficients as a function of the transverse motion and this is discussed in section 5.4.1 and compared to the results presented in this thesis.
Kim et al. (1986) report another experimental campaign performed later on by Prof. Vandiver’s team at St Croix. It consisted of field experiments on two test cables subjected to vertically sheared flow. One of the cables was 614 m in length with an external diameter of 0.406 cm and the second model was 2743 m with a diameter of 0.239 cm. They found infinite string behaviour with damped traveling waves along the length of the pipe. Very broadband spectra were found and single mode lock-in was not observed. The higher RMS responses were found were the current had higher speeds, with peak amplitudes ranging between 0.25 and 0.5 diameters. Mean drag coefficients were lower (around 1.5) than in cables subject to uniform flow under lock-in conditions, as reported in Vandiver (1983).

With the information obtained from these two experimental campaigns with long flexible cylinders, Vandiver (1993) proposed a set of parameters with the intention to predict lock-in under specific flow and structural conditions. He states that several reasons are the cause for lock-in not to appear in the response of a flexible structure: damping values big enough to avoid motion, shedding excitations far away from any natural frequency of the structure and the opposite possibility, the excitation bandwidth covers several natural frequencies resulting in a multi-mode and multi-frequency random response. He comments on the importance of the mass ratio, pointing out that cylinders with low mass ratios present a broader lock-in range than those with high mass ratios, figure 2.9 a fact also observed in rigid cylinder studies. Figure 2.9 shows response data for different mass ratios and how the reduced velocity ranges vary. The explanation given to the widening effect due to the lower mass ratios is that the cylinder resonant frequency is not constant inside the lock-in range, in fact it increases with increasing flow speed due to a decrease in the added mass, allowing the lock-in to persist over a broader reduced velocity range. Lock-in results in high responses and high drag coefficient values, therefore is considered to be the worst design case.

When dealing with sheared flows, one should expect as many frequencies as potentially the shear can input into the system and questions like which ones are going to control the motion become crucial. It is supposed that those frequencies existing because of the vortex shedding and close to the natural frequencies of the structure, would give
rise to a source of excitation. Vikestad (1998) carried out experiments with a segment of rigid cylinder towed in a flume, in order to identify how the excitation produced by the vortex shedding generated by a uniform flow, was affected by excitations at another frequencies. He carried out tests in uniform flow with a cylinder able to vibrate in the transverse direction, verifying previous work from other authors. He also conducted another set of tests where he introduced motions through the supports of his model, at different frequencies to those produced by the vortex shedding to simulate the excitation coming from vortex shedding generated at other parts of a riser pipe because of the shear. Comparing the results from both set-ups he concluded that the VIV response started at a reduced velocity of 4 independently of the support motions. He also observed that the amplitudes of the response were slightly lower in the disturbed supports case and it was decreasing with increasing support motion.

In Vandiver & Marcollo (2003) a discussion about the maximum attainable mode number is presented, with a compilation of dimensionless parameters relevant to analyzing VIV in long flexible cylinders. The role of added mass in a flexible cylinder is discussed and it is stated that the added mass decreases dramatically with increasing reduced velocity inside the lock-in range, producing an increase in the natural frequencies of the
cylinder. It is suggested that the lock-in regions are broad with respect to a sheared flow case because the added mass changes the natural frequencies in a way that it follows the vortex shedding frequency making the lock-in persist. In this paper data from an experiment on a pin-ended long flexible cylinder carried out by ExxonMobil is presented. The test model consisted of a flexible cylinder with an aspect ratio of about 83 subject to a tension around 16.7 kN giving natural frequencies in air near 2 Hz and critical damping ratios in the range of 1 to 1.5%. They show only the response of the cylinder in the first and the second mode of vibration. They show an overlap in the reduced velocities (based on the fundamental natural frequency in air) in the first and second mode responses, with both modes coexisting. The lock-in of the first mode is characterized by an almost linear increase of amplitude with reduced velocity (from 3 to 9.5) up to a point where the second mode becomes dominant. This effect and the mode overlapping, specially at high mode numbers, can be also observed in the data presented in chapter 5.4.1 of this thesis.

Lyons et al. (2003) present results from service measurements on the Foinaven umbilical, comparing their data with results from SHEAR7 (Vandiver 2003) predictions. They comment that the main differences between umbilicals and vertical tension risers are due to their complex geometries. Buoyancy modules provide zones with irregular diameters and increased damping, reducing VIV response or restricting the VIV to low frequencies. They found that in dynamic umbilicals the modes of interest are much higher than in vertical tension risers and are unlikely to include responses near the fundamental, and that single frequency response is unlikely to happen. They conclude that all these facts result in lower maximum amplitude responses when compared to vertical risers. They also indicate the inability of the codes to deal with varying parameters along the length such as mechanical properties and added mass effects.

de Wilde & Huijsmans (2004) performed experiments on a test pipe with an aspect ratio of 767, which was towed horizontally under tension. The fundamental natural frequencies of the pipe were about 1.5 Hz for a tension of 1 kN. The tests were carried out in the sub-critical regime for Re slightly higher than the ones presented later on in this thesis. They comment on the drag amplification due to the VIV lock-in process.
suffered by the pipe, leading to values of drag coefficient ranging between 1.5 and 2.7.

In Facchinetti et al. (2004a) the basis of VIV modeling by coupling Van der Pol wake oscillators with structural models of flexible pipes is presented. Results for shear flow cases are also presented. In another paper (Facchinetti et al. 2004b) the same authors study the effect of vortex induced traveling waves in a flexible cylinder. They use their wake oscillator model and compare the results with data obtained in their lab. A long flexible cylinder with an aspect ratio of 250 and a mass ratio of 1.6 is used for the purpose. The cable is towed in still water only from the top end, letting the lower end be unrestrained in order to model a partially non-reflecting boundary condition. They found the motion of the cable was governed by a single harmonic component.

Trim et al. (2005) describe a set of experiments which characterize the in-line and cross-flow response of a long flexible cylinder under uniform and shear flow profiles. A bare cylinder was tested as well as a straked model with different coverages (41, 62, 82, 91%). The authors did not measure forces on the models, so the analysis is focused purely on the response, the modal contribution to the overall response and in the comparison between the different coverage cases for each flow profile. An \( L/D \) of around 1400 was used with tensions varying between 4 and 6 kN and a mass ratio of approximately 1.6. Two different strake configurations were used, one with a pitch/diameter ratio of 5 and height/diameter ratio 0.14 and another one with 17.5 and 0.25 respectively. The experiment allowed the researchers to resolve modes up to the 24\(^{th}\) cross-flow and the 40\(^{th}\) in-line. They comment on the fact that the in-line fatigue is as important as the cross-flow. They show how in shear flow profiles, the VIV response is suppressed with strake coverages greater than 82% for strake geometries with 17.5D and 5D pitch. In uniform flows they found that only the 17.5D pitch with coverages greater than 82% was able to suppress VIV.

In order to investigate the effect of shear in a low flexural stiffness cylinder, Marcollo & Hinwood (2006) used a pin-ended flexible model 3.58 m in length with an external diameter of 40 mm \( (L/D=89.5) \). They used the flume at Monash University to generate a stepped flow consisting of two separate regions. Inside each region the flow was
uniform and one had greater velocity than the other. They reached Reynolds numbers in the range between 8000 and 40000. Excitations up to the second mode cross-flow and the third in-line were observed. Figure 2.10 shows the amplitude response observed and how at a reduced velocity of around 8, the second mode (C.F. n=2 in the figure) takes over from the first (C.F. n=1 in the figure) in controlling the response. In-line dominant frequency to shedding frequency ratios of up to 4 are found. The authors conclude that depending on the power input configuration of the shear, the cylinder may exhibit single mode lock-in or multi-mode behaviour.

![Figure 2.10: Cross-flow and in-line response amplitude vs. reduced velocity, from Marcollo & Hinwood (2006)](image-url)

A paper by Lie & Kaasen (2006) is focused on the modal analysis technique used to obtain the modal contributions from strain gauge measurements, which is the same as the one used in this thesis. The cylinder had a mass ratio of 3.13, a length of 90 m with an outer diameter of 0.03 m and the towing speeds ranged from 0.16 to 2 m/s.
The highest cross-flow excited mode was expected to be the 33\textsuperscript{rd} and the 50\textsuperscript{th} in-line. They show results of 13 tests cases of a long flexible cable subject to shear flow. They found a Strouhal number of about 0.17 and a quite constant cross-flow/in-line motion of around 5. They show similar values of curvature in-line and cross-flow supporting the idea that in order to analyze the fatigue life of a riser the in-line curvatures need to be taken into account. They report very broadband spectra of the strain signals without any evidence of lock-in behaviour.

2.4.3.2 Prediction methods

Prediction methods can be divided into two types: those based on experimental data, the empirical or semi-empirical codes and those based on CFD solvers coupled to structural models. Two-dimensional CFD algorithms in strip theory codes are faster and less computer demanding than 3D flow solvers. Regarding the structural modeling, the main differences between the models are the inclusion of non-linearities (geometrical and material non-linearities are the most common) or not in the calculation process. Depending on the specific problem being dealt with it can be necessary or not. The complexity increases the cost and the needed computing resources making sometimes the codes inapplicable for industrial purposes.

A brief description of the most used codes can be found in Chaplin et al. (2005). The paper shows the results of a blind prediction exercise in which some experimental runs presented later on in this thesis (chapter 5), are compared against the output of eleven numerical models. An earlier paper by Larsen & Halse (1995) presents a comparison of seven codes. In this case, the results where not compared against any experimental data. Some of the codes that appeared in this earlier paper appear again in the blind prediction exercise. The later comparison exercise shows better agreement between the different codes.

George Karniadakis and his team at Brown University have been using the three-dimensional DNS code NekTar, an spectral/hp finite element technique.
& Sherwin 1999), to compute the flow around rigid and flexible cylinders, coupled to appropriate structural models of pin-ended cylinders or cylinders with periodic ends. They have been researching on the vortex structures, the fluid loading and the response on different aspect ratio cylinders, at low Reynolds numbers. Newman & Karniadakis (1996) show results for two types of cylinders restricted to move cross-flow, the first with an aspect ratio of 12.6 (two Re numbers: 100 and 200) and the second one with 45 (three Re numbers: 100, 200 and 300). Response, lift and drag forces are computed for both cases under two distinct situations: flow-induced and forced vibrations. With the low aspect ratio cylinder at the low Reynolds number they observed primarily standing wave behaviour with cross-flow amplitudes around 0.7. The lift values are similar between the free and the forced vibration simulations and the drag is amplified 40% with respect to that expected on a stationary cylinder at a Re=100. At Re=200 the behaviour is dominated by traveling waves with amplitudes up to 1 diameter and higher amplification of the drag is seen with similar values of lift. Similar observations are made regarding the high aspect ratio cylinder but with differences in the wake structures. An extended study with more details about the flow structures in the wake was presented by the same authors in the Journal of Fluid Mechanics (Newman & Karniadakis 1997) and Evangelinos & Karniadakis (1999).

Evangelinos et al. (2000) used NekTar to determine fluid forces acting on a cylinder responding in its second mode: in one case as a standing wave and in the other as a traveling wave. In their paper they present distributions of predicted instantaneous fluid forces and comment on the need for benchmark experiments with lift and drag data that could be used to help validate their computational results. They analyze a rigid cylinder and several types of flexible pipes, by defining periodic or fixed ends and by using low ($L/D \approx 12$) or high aspect ratios ($L/D \approx 378$), at a Re=1000. In the case with periodic ends, they saw traveling waves dominating the response even though a standing wave was used as the initial condition for the computation. Higher amplitudes are registered when the cylinder vibrates in a standing wave fashion and the observed maximum lift coefficients under this situation are about 2 for the short beam and up to 3.5 for the long one. Even though the beam responds in its second mode
(figures 2.11(a) to 2.11(c)), the observed drag coefficients are 3 times higher (around 3.5 or 4) than those typical of stationary cylinders at the same Re. They compare their time averaged force values, with previous authors, finding quite good agreement with their computations, even though different Reynolds numbers were used. Finally they propose an expression for the drag coefficient as a function of the cross-flow motion. This expression is checked against the experimental data presented in this thesis, in chapter 5.

Yamamoto et al. (2004) used strip theory with a Lagrangian numerical technique, the discrete vortex method, to compute 2D incompressible viscous flow coupled to a FEM model of a vertical riser pipe based on the Euler-Bernoulli equations. They present results for a flexible cantilever and for a single vertical tension riser. The cantilever results are compared to experimental results obtained by Condino-Fujarra. (2002). The flexible pipe results are computed using 40 2D sections along the length of the pipe to compute the fluid loading and to couple them to the structural model and they are compared against other numerical results presented by Ferrari (1998), based on a quasy-steady model in which the forces are calculated using the Morison equation and a quasy-steady model for the lift force, then coupled to a similar structural FEM code (Ferrari & Bearman 1999). Peak amplitudes up to 0.6 diameters are found in modes from the fundamental to the fourth, with reduced velocities based on the dominant oscillation frequency ranging between 4 and 7. Strouhal numbers vary between 0.15 and 0.22, with very clearly defined spectra peaks corresponding to the shedding frequencies. In the case of sheared flow profiles the single frequency responses observed in the uniform flow are no longer evident and several frequencies define the structural response. The comparisons show very good agreement producing very similar in-line and cross-flow envelopes.

Willden & Graham (2001) present results on the flow-induced transverse vibrations of a rigid two-dimensional low mass cylinder, elastically mounted with zero damping, and on the transverse vibrations of a long flexible cylinder in sheared flow. They use strip theory with a two-dimensional hybrid Eulerian/Lagrangian Navier Stokes code to compute the flow at different positions along the length of the riser. The
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(a) Spatio-temporal plot of the amplitude. Low aspect ratio beam, from Evangelinos et al. (2000)

(b) Spatio-temporal plot of the lift coefficient. Low aspect ratio beam, from Evangelinos et al. (2000)

(c) Spatio-temporal plot of the drag coefficient. Low aspect ratio beam, from Evangelinos et al. (2000)

Figure 2.11: Results from Evangelinos et al. (2000)
CHAPTER 2. Literature review

Hydrodynamic link between each 2D sections is made through a 3D large-scale vortex lattice representation of the wake. The flow is then linked to a finite element model of the riser pipe. The flexible cylinder simulation is of a long cylinder with an aspect ratio of 100 subject to a shear with a maximum Re=200. A mass ratio slightly over 4 with a top tension high enough to excite the fundamental mode as desired. Figure 2.12 illustrates the response and the spatio-temporal evolution of the lift coefficient and the Strouhal along the length of the riser. Lift and motion appear to be in phase in the lower part of the riser \((z/D < 44)\) and in antiphase in the upper part, and despite the shear, the shedding is correlated along the length except in the near-end regions.

Figure 2.12: Spatio-temporal plot of the lift coefficient and evolution of Strouhal number along the length of a flexible riser \((L/D=100)\), from Willden & Graham (2001)

In Willden & Graham (2004) the same authors use their code VIVIC (Willden 2003) to study the transverse Vortex-Induced Vibrations of a long \((L/D=1544)\), flexible pipe subjected to a uniform current profile with a Reynolds number of \(2.84 \cdot 10^5\). The mass ratio is varied between 1.0 and 3.0 by changing the density of the internal fluid modeled as a part of the SCR system. They observed multi-mode behaviour with all excited modes vibrating at the same frequency (that expected from the Strouhal relationship) even though the approaching flow was set to be uniform. They show in their work that as the mass ratio is increased the natural frequencies are reduced and become more
closely spaced. They comment on the effect of the added mass and how it is able to force a vibration at a frequency far from the natural frequencies of the structure.

The study of the past literature reveals that there is still a lack of data for long flexible cylinders undergoing VIV. The difficulties when designing test campaigns are usually related to the limiting size of the laboratory facilities, the available instrumentation and the impossibility to achieve realistic Reynolds numbers. In laboratory measurements the researchers can control the main parameters, but in field experiments they cannot be controlled, therefore the interpretation of the results needs to be carefully made. The vortex structures in the wake of long flexible cylinders have not been investigated for high aspect ratio cylinders undergoing high mode VIV. It would give invaluable information when compared to the response and the fluid loads generating the response. Suppression devices need to be studied in more detail in the near future. Helical strakes have been tested and compared to bare cylinders demonstrating good performance but more work needs to be done in order to understand how they really work.

Even though the prediction tools have shown an improvement in recent years, faster and more reliable codes are needed for industry design purposes. Empirical codes are fast and easy to use, but they rely on the scarce available data available. They are incapable to model accurately suppression devices. CFD tools are complicated to use and very demanding computationally, therefore not practical nowadays for design purposes. The possibility to model suppression devices is a need for the industry and together with the evolution of the computational resources in the next years, it will play an important role in the design stages.
Chapter 3

Mathematical modeling of vertical tension risers

In this section the governing equations for a vertical pin-end beam subject to a varying tension, are derived. In beams with high aspect ratio such as riser pipes, Euler-Bernoulli beam theory can be used to model the dynamics, because the transverse shear can be neglected. When the diameter or width of the cross-section is small compared to the length of the beam, it can be considered that the planes perpendicular to the axis remain plane and perpendicular to the axis after the deformation. In cases where this effect cannot be neglected, Timoshenko beam theory is more convenient. Euler-Bernoulli equation can be applied to the riser model used in the experiments described in section 4 by using the Finite Element Method. Extensive literature about the use and about the theory, computer implementation and programming of the FEM is available, see Bathe (1996), Reddy (1993), Reddy (2005) and Masdemont-Soler (2002).

A FEM code was written specifically as a part of this research work, mainly with two objectives. The first was to be able to obtain the natural frequencies and mode shapes needed to analyze the data obtained in the experiments described in 5. This can be done by solving the eigenvalue problem associated with the governing equations, section 3.3. The second objective was to investigate the hydrodynamic forces exerted
on the riser model by inputting the experimental data obtained into the FEM code, see chapter 6.

3.1 Governing equations

The vertical tension marine riser described in 4 can be idealized as a beam with low flexural stiffness. In operating risers, the hydrostatic pressure due to the inner circulating fluids has an effect on the effective tension variation along the length of the riser, and it must be modeled. Top end motions due to surface wave impacts and motions of the platforms are also important to precisely describe the dynamics of such systems. Mechanical properties are normally not uniform due to the existence of joints and other mechanical devices, which produce variations of the flexural and axial stiffness along the length of the pipes. All these factors make the structural dynamical modeling of riser pipes very complex. Considerably simplifications can be done to the dynamical model if factors such as those stated above are not taken into account. It is similar to what happens in a laboratory test campaign such as the one described in this thesis. All these aspects were specifically avoided to produce data obtained from a situation in which the main parameters were totally under control, focusing the research in the fundamental aspects of the multi-modal VIV response and loading on vertical risers with linearly varying tension.

A cartesian reference with its origin at the bottom of the riser has been used, in which the $x$ axis is parallel to the flow velocity, $z$ coincides with the vertical axis of the riser in its undeflected configuration and the $y$ is perpendicular to both. $u(z, t)$, $v(z, t)$ and $w(z, t)$ are defined as the time variant in-line, cross-flow and the axial motions respectively. With these set of displacements a point in the centerline of the beam can be spatially described. Combinations of translations and rotations around the beam axis describe states of torsion, and the states of bending are described by displacements and rotations around the two axes contained in the plane perpendicular to the beam axis. Because the riser was attached to the supporting structures at its ends with
universal joints, torsion motions were avoided, but the fact that the riser model was free to move in-line and cross-flow at the same time, meant that small twisting motions were inevitable. A study was performed by Prof. John Chaplin at Southampton University in the experiment design stage, in which the effects of torsion were found to be neglectable, see Appendix A in Chaplin et al. (2005). The projections of the out-of-plane movements into the $xz$ and $yz$ planes were calculated, using a commercial FEM structural code in which the load applied to the riser consisted of a steady in-line drag distribution and a steady sinusoidal cross-flow lift force, considering the twisting and without considering it. The results were almost identical and the misalignment introduced by the twisting, in the strain gauge measurements was in the worst case less than 0.6 degrees.

Different non-linearities can be considered if necessary, and included in the dynamical models that describe the response of the riser pipes. If the riser operates inside its elastic limit the material non-linearity does not play a role in the dynamics and it can be neglected. The riser model used was equipped with an array of springs at its top end, to avoid reaching its elastic limit due to the increase in tension produced by the fluid loading. It was forced to respond inside its linear elastic range and non-linear material properties were avoided that way. Geometric non-linearities are important when large deflections occur introducing extra terms in the stress equations due to the effect of moments coming from the misalignment of the forces. When the first derivative in space of the transverse motions is small ($\frac{\partial u}{\partial z} \approx 0$ and $\frac{\partial v}{\partial z} \approx 0$), as in the case presented here, the non-linear terms in the stress equations disappear and both transverse and axial motions become uncoupled, (Reddy 1993) and (Reddy 2005). Therefore, in problems involving small displacements the motions in the the planes $xz$ and $yz$ can be treated independently.

3.1.1 Equation for the transverse motion

The Euler-Bernoulli or classical theory for beams assumes that (Masdemont-Soler 2002):
− Planar surfaces orthogonal to the axis of the beam, remain planar and orthogonal to the axis after the deformation. Therefore transverse shear is neglected.

− All the forces acting on the beam can be expressed by means of vectors parallel to the $x$ or $y$ axis (except the axial forces in our case of study).

− The transverse section of the beam is symmetric with respect to plane $xz$ or $yz$.

Then the transverse deformation of a generic beam can be described with a fourth order differential equation which can be derived by applying force and momentum equilibrium to an infinitesimal section of the beam, see the free-body diagram in figure 3.2. The formulation can be similarly applied to any of the two transverse directions, $x$ or $y$.

$$
\left( \tau(z,t) + \frac{\partial \tau(z,t)}{\partial z} dz \right) - \tau(z,t) - f(z,t)dz + f_i(z,t)dz = 0
$$

Figure 3.1: Cartesian axis reference system
where \( f \) is the external fluid force, \( f_i \) is the inertial force and \( \tau \) is the shear force, all referred to one of the transverse directions, \( x \) or \( y \). In the case of the cross-flow direction, \( v(z,t) \) would be used. Dividing all terms by \( dz \), then

\[
\frac{\partial \tau(z,t)}{\partial z} = f(z,t) - f_i(z,t)
\]  

(3.1)

With the inertia force described as follows:

\[
f_i(z,t) = \rho(z)A(z) \frac{\partial^2 u(z,t)}{\partial t^2} = m(z) \frac{\partial^2 u(z,t)}{\partial t^2}
\]

where \( \rho(z) \) is the density of the beam, \( A(z) \) is the cross sectional area of the beam and \( m(z) \) is the mass of the beam per unit length.

On the other hand, equilibrium of moments in the beam element gives us:

\[
- \left( M(z,t) + \frac{\partial M(z,t)}{\partial z} dz \right) + \tau(z,t)dz + M(z,t) + T(z) \frac{\partial u(z,t)}{\partial z} dz = 0
\]

dividing by \( dz \) we obtain

\[
\tau(z,t) = \frac{\partial M(z,t)}{\partial z} - T(z) \frac{\partial u(z,t)}{\partial z}
\]
The first derivative of $\tau(z,t)$ respect to $z$ is

$$\frac{\partial \tau(z,t)}{\partial z} = \frac{\partial^2 M(z,t)}{\partial z^2} - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) $$

Substituting (3.1) in (3.2) one can obtain

$$\frac{\partial^2 M(z,t)}{\partial z^2} - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) + m(z) \frac{\partial^2 u(z,t)}{\partial t^2} = f(z,t)$$

and substituting in the above equation $M(z,t) = EI(z) \frac{\partial^2 u(z,t)}{\partial z^2}$ we obtain the differential equation that governs the transverse deflection of a beam with flexural stiffness $EI(z)$ and with an applied tension $T(z)$. This equation has been obtained neglecting the effects of rotational inertia and damping. The modeling of damping will be discussed in section 3.4.

$$\frac{\partial^2}{\partial z^2} \left( EI(z) \frac{\partial^2 u(z,t)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) + m(z) \frac{\partial^2 u(z,t)}{\partial t^2} = f(z,t)$$

The boundary conditions for the above differential equation in the case of a pin-ended beam are:

$$u(0,t) = 0, \quad u(L,t) = 0 \quad \forall t$$

$$\frac{\partial^2 u(0,t)}{\partial z^2} = 0, \quad \frac{\partial^2 u(L,t)}{\partial z^2} = 0 \quad \forall t$$

The equation above includes the effect of axial tension and can be used to describe the deflection of the riser model used for the experiments. This equation is a fourth order partial differential equation with a time variant term, and it can be can be rewritten as
\[ EI \frac{\partial^4 u(z,t)}{\partial z^4} - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) + m \frac{\partial^2 u(z,t)}{\partial t^2} = f(z,t) \] (3.4)

assuming that the mass \( m(z) = m \) and the flexural stiffness \( EI(z) = EI \) are uniform along the length of the riser due to the design of the model. The tension in equation 3.4 can be expressed as

\[ T(z) = T_t - w_s (L - z) \] (3.5)

with \( T_t \) being the tension applied at the top of the riser, \( L \) the length of the riser and \( w_s \) the submerged weight per unit length. The equation considers the effect of buoyancy on the riser because the submerged weight is used and considered to be constant.

### 3.1.2 Equation for the axial motion

Applying the same assumptions as in section 3.1.1 the equations for the axial motion can be derived.

\[ \frac{\partial T(z,t)}{\partial z} + f_z(z,t) = m \frac{\partial^2 w(z,t)}{\partial z^2} \]

The definition of strain allows us to derive the expression of the tension relating it to the axial motions. The general expression for the axial strain can be found in (Reddy 2005). In the present work, it will be referred to from now on as \( \epsilon \), being

\[ \epsilon = \frac{\partial w}{\partial z} + \frac{1}{2} \left( \frac{\partial u}{\partial z} \right)^2 \]

If there are not large displacements, geometric non-linearity coming from these terms can be neglected because \( \frac{\partial w}{\partial z} \approx 0 \) and \( \frac{\partial u}{\partial z} \approx 0 \). Therefore, \( \epsilon \) becomes

\[ \epsilon(z,t) = \frac{\partial w(z,t)}{\partial z} \] (3.6)
The stress is related to the strain by means of the Elastic modulus,

\[ \sigma(z, t) = E \varepsilon(z, t) = E \frac{\partial w(z, t)}{\partial z} \]  

(3.7)

and the tension is related to the stress through the cross-sectional area of the beam,

\[ T(z, t) = A \sigma(z, t) = EA \frac{\partial w(z, t)}{\partial z} \]  

(3.8)

In this case, because of its relation to the axial strain, the tension is a function not only of \( z \) but also of time. Hence, the equation for the axial motion of the beam is

\[ m \frac{\partial^2 w(z, t)}{\partial t^2} - EA \frac{\partial^2 w(z, t)}{\partial z^2} = f_z(z, t) \]  

(3.9)

with the following boundary conditions

\[ w(0, t) = 0 \quad \forall t \]  

(3.10)

Note that the axial displacement at the top of the riser model is not zero, because the attachment of the model, was made through an array of springs that allowed vertical motion, as it appears in figure 3.1. This was done, as it will be explained in chapter 4, to avoid exceeding the elastic limit of the material when the tension applied to the top increased due to the drag generated by the current.

### 3.2 The Finite Element method (FEM)

There are several ways of numerically solving partial differential equations that have a complex analytic solution. A classification of these methods could be (Masdemont-Soler 2002), (Reddy 1993):
CHAPTER 3. Mathematical modeling of vertical tension risers

▷ Finite Difference methods.

▷ Variational methods.

▷ Spectral methods.

One of the most extensively used variational method is the Finite Element Method. The method is based on:

1. Discretization of the domain of the governing equation into small parts called Finite Elements.

2. Transformation of the governing equation in an integral form (weak formulation) because integral equations are easier to solve numerically.

3. Approximation of the solution as a linear combination of special functions in each element of the domain.

4. Assembly of the approximations for each element to obtain the global system.

5. Time discretisation.

After these steps the initial partial differential equations are transformed into an algebraic system much easier to solve.

3.2.1 Discretisation of the riser

The riser pipe is idealised as a one-dimensional domain coincident with the neutral axis of the structure. A mesh formed by one-dimensional elements can be used, in which a generic finite element $\Omega^e$ consists of two nodes, and is referred to, as

$$\Omega^e = [z_1^e, z_2^e]$$

with the length of the element defined as
being \( z^e_1 \) and \( z^e_2 \) the global coordinate of the finite element at the first and the second node respectively. The number of elements \( n_e \) is

\[
n_e = \frac{L}{h^e}
\]

and the number of nodes

\[
n = n_e + 1
\]

The solution of the equation (3.11) in each element will be approximated with

\[
u^e(z, t) \simeq u^e(t) \Psi^e(z) = \sum_{j=1}^{N_t} u^e_j(t) \psi^e_j(z) \tag{3.11}
\]

\[
v^e(z, t) \simeq v^e(t) \Psi^e(z) = \sum_{j=1}^{N_t} v^e_j(t) \psi^e_j(z) \tag{3.12}
\]

\[
w^e(z, t) \simeq w^e(t) L^e(z) = \sum_{j=1}^{N_a} w^e_j(t) \ell^e_j(z) \tag{3.13}
\]

According to this approximation, the transverse deflections of the riser in each finite element, depending on time and position, \( u^e(z, t) \) and \( v^e(z, t) \), are the linear combination of the time dependent deflections at the node \( j \), that is \( u^e_j(t) \) and \( v^e_j(t) \), multiplied by spatial approximation functions \( \psi^e_j(z) \), according to each of the \( N_t \) degrees of freedom in both nodes of the finite element. The same applies to the axial motions \( w^e_j(t) \) with the spatial functions \( \ell^e_j(z) \)s and \( N_a \) degrees of freedom. The approximation functions are known as shape functions.

The bending states result from motions and rotations (first derivatives in space of the displacements), so the finite elements for the transverse case must be able to represent
two degrees of freedom at each node, and it implies \( N_t = 4 \) degrees of freedom. The rotations around the axis of the pipe are neglected as stated in 3.1, and this results in only one degree of freedom at each node of the finite element associated to the axial displacements with \( N_a = 2 \) degrees of freedom. Equations 3.11 to 3.13 can be rewritten in a collective manner as follows,

\[
\begin{align*}
  u^e(z, t) & \simeq (P^e_x r^e) \Psi^e \\
  v^e(z, t) & \simeq (P^e_y r^e) \Psi^e \\
  w^e(z, t) & \simeq (P^e_z r^e) L^e 
\end{align*}
\]

where the displacement vector \( r^e \) which includes \( u^e, v^e \) and \( w^e \), is

\[
\begin{bmatrix}
  u^e_1 \\
  u^e_2 \\
  v^e_1 \\
  v^e_2 \\
  w^e_1 \\
  u^e_3 \\
  u^e_4 \\
  v^e_3 \\
  v^e_4 \\
  w^e_2
\end{bmatrix}^T
\]

The \( P^e \) matrices map the degrees of freedom with the corresponding displacements or rotations in the elemental displacement vector \( r^e \). The only non-zero elements in \( P^e_x \) are the ones occupying positions \([1,1], [2,2], [6,6]\) and \([7,7]\), which are 1’s. In \( P^e_y \) the 1’s are located at the \([3,3], [4,4], [8,8]\) and \([9,9]\) positions, whilst in \( P^e_z \) the 1’s are in \([5,5]\) and \([10,10]\). \( P^e_x \) is shown below as an example.

\[
P^e_x = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
This notation is very helpful when constructing the elemental mass and stiffness matrices in section 3.2.2.

In the following, only the results of the application of the method are presented, in a way that would allow any other interested party to code in a computer program the matrices found here. The complete derivation of the FEM through the weak formulation of the governing equations and the shape functions is presented in appendix A.

3.2.2 Elemental matrices

3.2.2.1 Transverse motion elemental matrices

Substituting the interpolation polynomials as the weight function \( \vartheta(z) \) in the weak formulations, equations 3.1.1 and 3.9, and using the approximations 3.11, 3.12, one obtains for the transverse cases:

\[
\int_{z_1^e}^{z_2^e} \left[ T(z) \frac{\partial \psi_i^e}{\partial z} \left( \sum_{j=1}^{N_t} u_j^e \frac{\partial \psi_j^e}{\partial z} \right) + EI \frac{\partial^2 \psi_i^e}{\partial z^2} \left( \sum_{j=1}^{N_t} u_j^e \frac{\partial^2 \psi_j^e}{\partial z^2} \right) + m \psi_i^e \left( \sum_{j=1}^{N_t} \psi_j^e \frac{\partial^2 u_j^e}{\partial t^2} \right) - w_f \right] dz
\]

- \( Q_1^e(z_1^e) \) - \( Q_2^e(z_2^e) \) - \( Q_1 \left( \frac{\partial \psi_i^e}{\partial z} (z_1^e) \right) \) - \( Q_2 \left( -\frac{\partial \psi_i^e}{\partial z} (z_2^e) \right) = 0 \quad (3.19) \)

with the secondary variables being

\[
Q_1(t) = \left[ -T(z) \sum_{j=1}^{N_t} u_j^e \frac{\partial \psi_j^e}{\partial z} + \frac{\partial}{\partial z} \left( EI \sum_{j=1}^{N_t} u_j^e \frac{\partial^2 \psi_j^e}{\partial z^2} \right) \right]_{z=z_1^e} \quad (3.20)
\]

\[
Q_2(t) = -\left[ -T(z) \sum_{j=1}^{N_t} u_j^e \frac{\partial \psi_j^e}{\partial z} + \frac{\partial}{\partial z} \left( EI \sum_{j=1}^{N_t} u_j^e \frac{\partial^2 \psi_j^e}{\partial z^2} \right) \right]_{z=z_2^e} \quad (3.21)
\]

\[
Q_1(t) = EI \sum_{j=1}^{N_t} u_j^e \frac{\partial^2 \psi_j^e}{\partial z^2} \bigg|_{z=z_1^e} \quad (3.22)
\]

\[
Q_2(t) = EI \sum_{j=1}^{N_t} u_j^e \frac{\partial^2 \psi_j^e}{\partial z^2} \bigg|_{z=z_2^e} \quad (3.23)
\]
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Equation (A.1) results in a system of algebraic equations that gives the approximate solution of (3.4) in the generic element $\Omega_e$. It is the algebraic equation for both transverse motions, either $x$ or $y$. It can be expressed in matrix form as

\begin{align}
In - line & \quad K^e U^e + M^e \ddot{U}^e = F_x^e + Q_x^e \quad (3.24) \\
Cross - flow & \quad K^e V^e + M^e \ddot{V}^e = F_y^e + Q_y^e \quad (3.25)
\end{align}

where the vectors $U^e$ and $V^e$ give the displacements along the axis of the cylinder in the transverse directions. The double dot indicates second derivative in time. The matrices are $[4x4]$ because there are two nodes and at each of them, two degrees of freedom. Note that the displacement vector $U^e$ is formed by $u_i$ and $-\frac{\partial u}{\partial z}$ at each node, then its components are $U^e = [u_1 u_2 u_3 u_4]^T$. $u_1$ is the displacement at the first node ($z^e_1$), $u_2 = -\frac{\partial u}{\partial z}$ is the rotation in the first node, $u_3$ is the displacement at the second node ($z^e_2$) and $u_4 = -\frac{\partial u}{\partial z}$ the rotation at the second node. $K^e$ is the stiffness matrix, $M^e$ is the consistent mass matrix, $F^e$ is the transverse nodal forces vector, and finally $Q^e$ is the secondary variables vector. The reader must notice that $K^e$ and $M^e$ are valid for any of the two transverse directions, $x$ or $y$. They are both symmetric and calculated as follows,

\[ K^e = K^e_1 + K^e_2 \]

The components of the matrices are

\begin{align}
K^e_{1ij} &= K^e_{1ji} = \int_{z^e_1}^{z^e_2} T(z) \frac{d^2 \psi_i(z)}{dz^2} \frac{d^2 \psi_j(z)}{dz^2} dz \quad (3.26) \\
K^e_{2ij} &= K^e_{2ji} = \int_{z^e_1}^{z^e_2} EI \frac{d^2 \psi_i(z)}{dz^2} \frac{d^2 \psi_j(z)}{dz^2} dz \quad (3.27)
\end{align}
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\[ M_{ij}^e = M_{ji}^e = \int_{z_i^e}^{z_j^e} m\psi_i(z)\psi_j(z)dz \]  
\[ F_i^e = \int_{z_i^e}^{z_j^e} f(z,t)\psi_i(z)dz \]  

Expressed in terms of the local generic element \( \Omega^R = [-1, 1] \) (see section A.3), are

\[ K_{ij}^R = K_{ji}^R = \int_{-1}^{1} \frac{2T^R(\zeta)}{h^e} \frac{d\psi_i^R(\zeta)}{d\zeta} \frac{d\psi_j^R(\zeta)}{d\zeta} d\zeta \]  
\[ K_{ij}^{R2} = K_{ji}^{R2} = \int_{-1}^{1} \frac{8EIh^e}{\psi_i^R(\zeta)} \frac{d^2\psi_j^R(\zeta)}{d\zeta^2} d\zeta \]  
\[ M_{ij}^R = M_{ji}^R = \int_{-1}^{1} \frac{mh^e}{2} \psi_i^R(\zeta)\psi_j^R(\zeta) d\zeta \]  
\[ F_i^R = \int_{-1}^{1} \frac{h^e}{2} f(z,t)\psi_i^R(\zeta) d\zeta \]

For the case of constant \( T \), the matrices resulting from equations 3.30 to 3.33 can be obtained by solving analytically the integrals above. The matrices do not change in the different finite elements, they only depend on the element height \( h^e \) and they can be found in any introductory FEM book, (Masdemont-Soler 2002) or (Reddy 1993). If \( T \) cannot be considered constant along the axis of the riser, the matrices change for the different finite elements forming the mesh and they must be calculated specifically for each one. The same happens with the mass matrix. The elemental mass matrix changes if there are changes of mass along the axis of the riser. In the experiments described in 4, different situations were tested, and that requires that in some cases mass changes need to be taken into account. The tension is considered in all cases a function of \( z \), \( T = T(z) \), as shown in (3.5). The integration of equations 3.30 to 3.33 for the case of variable tension is very complex analytically, and numerical integration needs to be used. Gauss-Legendre Quadrature formulae, as seen in section 3.2.2.3, have been used for this purpose, and \( T(z) \) has to be expressed in terms of the local coordinate \( T^R(\zeta) \),

\[ T^R(\zeta) = T_l + w \left( \frac{h^e}{2} + z_1^e - L \right) + \frac{wh^e}{2} \zeta \]  

65
so equation \(3.30\) transforms into a more complex expression

\[
K_{ij}^R = K_{ji}^R = \frac{h^e}{2} \left\{ \left( T_i + w \left( \frac{h^e}{2} + z_1^e - L \right) \right) \right. \\
\left. \int_{-1}^{1} \frac{d\psi_i^R(\zeta)}{d\zeta} \frac{d\psi_j^R(\zeta)}{d\zeta} d\zeta + \right. \\
\left. \frac{wh^e}{2} \int_{-1}^{1} \frac{d\psi_i^R(\zeta)}{d\zeta} \frac{d\psi_j^R(\zeta)}{d\zeta} d\zeta \right\} (3.35)
\]

It is very convenient for the implementation of the method to divide the stiffness \(K^e\) and mass \(M^e\) matrices into \([2\times2]\) dimension submatrices, which allows us to rewrite them in nodal form, to ease the assembly process described in the next section.

\[
K^e = \begin{bmatrix}
K_{a}^e & K_{b}^e \\
K_{c}^e & K_{d}^e
\end{bmatrix}
\]

where

\[
K_a^e = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}, \quad K_b^e = \begin{bmatrix}
K_{13} & K_{14} \\
K_{23} & K_{24}
\end{bmatrix}
\]

\[
K_c^e = \begin{bmatrix}
K_{31} & K_{32} \\
K_{41} & K_{42}
\end{bmatrix}, \quad K_d^e = \begin{bmatrix}
K_{33} & K_{34} \\
K_{43} & K_{44}
\end{bmatrix}
\]

(3.36)

taking into account that the same nomenclature is followed for the mass matrix \(M^e\).

### 3.2.2.2 Axial motion elemental matrices

The same procedure applied to the weak form of the axial motion governing equation, including the finite element approximations \(3.13\) and the axial shape functions \(A.13\) results in
\[
\int_{z_1^e}^{z_2^e} \left[ m \ell_1^e(z) \left( \sum_{j=1}^{N_a} \ell_j^e \frac{\partial^2 u_j^e}{\partial z^2} \right) - EA \left( \sum_{j=1}^{N_a} u_j^e \frac{\partial \ell_j^e}{\partial z} \right) \right] \frac{\partial \ell_i^e(z,t)}{\partial z} \, dz + \ell_i^e(z_1^e)Q_1^a - \ell_i^e(z_2^e)Q_2^a = 0 \quad (3.37)
\]

with
\[
Q_1^a(t) = EA \left( \sum_{j=1}^{N_a} u_j^e \frac{\partial \ell_j^e}{\partial z} \right) \bigg|_{z=z_1^e} \quad (3.38)
\]
\[
Q_2^a(t) = EA \left( \sum_{j=1}^{N_a} u_j^e \frac{\partial \ell_j^e}{\partial z} \right) \bigg|_{z=z_2^e} \quad (3.39)
\]

The equations can be written in matrix form as follows
\[
K_e^z W_e + M_e^z \ddot{W}_e = F_e^z + Q_e^z \quad (3.40)
\]

where \( W_e = [w_1 \ w_2]^T \), with its components the displacements at the first \((z_1^e)\) and second node \((z_2^e)\). By analogy to the transverse cases,

\[
K_e^{zi} = K_e^{zji} = \int_{z_1^e}^{z_2^e} \frac{d\ell_1^e(z)}{dz} \frac{d\ell_j^e(z)}{dz} \, dz \quad (3.41)
\]
\[
M_e^{zi} = M_e^{zji} = \int_{z_1^e}^{z_2^e} m \ell_1^e(z) \ell_j^e(z) \, dz \quad (3.42)
\]
\[
F_e^{zi} = \int_{z_1^e}^{z_2^e} f_z \ell_1^e(z) \, dz \quad (3.43)
\]

In terms of the local generic element \( \Omega^R = [-1, 1] \), they are

\[
K_e^{Rzi} = K_e^{Rzji} = \int_{-1}^{1} \frac{2EA}{h_R} \frac{d\ell_1^e(\zeta)}{d\zeta} \frac{d\ell_j^e(\zeta)}{d\zeta} \, d\zeta \quad (3.44)
\]
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\[ M_{zij}^R = M_{zji}^R = \int_{-1}^{1} \frac{mhR}{2} \ell_i^e(\zeta) \ell_j^e(\zeta) d\zeta \] (3.45)

\[ F_{zi}^R = \int_{-1}^{1} \frac{hR}{2} f_z^e(\zeta) d\zeta \] (3.46)

Because \( EA \), and \( m \) are constant along the length of the riser model, they do not vary for the different elements composing the mesh and are easy to solve analytically, resulting in,

\[ K_z^e = \frac{EA}{h^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad M_z^e = \frac{mh^e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad F_z^e = \frac{h^e f}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \] (3.47)

3.2.2.3 Computer implementation: Quadrature formulae

The Gauss-Legendre Quadrature formulae, given a number \( N \) of evaluation points, allows us to calculate exactly integrals of polynomials up to a certain order. The formulae has the expression

\[ \int_{-1}^{1} G(\zeta) d\zeta \simeq \sum_{j=1}^{N} W_j G(\zeta_j) \] (3.48)

The values of \( \zeta_i \) and \( W_j \) are tabulated according to the order of \( G(\zeta) \) and the number of evaluation points \( N \). Such tables can be found in any book of numerical analysis, (Masdemont-Soler 2002) or (Stoer & Burlish 1983).

The introduction of the local coordinate \( \zeta \) into the finite element equations is necessary to allow the use of the Quadratures producing fast and reliable computer implementations of the method. The integrals defined in equations 3.30 to 3.32 and 3.44 to 3.46 are solved with this method.
3.2.2.4 Generic element matrices

At this stage the elemental stiffness and mass matrices have been defined. The transverse matrices $K^e$ and $M^e$ are valid both for the cross-flow and the in-line motions. The axial case has been derived in parallel and the equations are shown in 3.40. Each element of the matrices can be calculated by means of the Gauss-Legendre Quadrature. The elemental matrices can be arranged collectively producing a $[10x10]$ matrix corresponding to the five degrees of freedom at each node, as in equations 3.14 to 3.16.

$$K^e = P_x^e K_x^e P_x^e + P_y^e K_y^e P_y^e + P_z^e K_z^e P_z^e$$  \hspace{1cm} (3.49)

$$M^e = P_x^e M_x^e P_x^e + P_y^e M_y^e P_y^e + P_z^e M_z^e P_z^e$$  \hspace{1cm} (3.50)

The $K_x^e$, $K_y^e$, $K_z^e$ and the $M_x^e$, $M_y^e$, $M_z^e$ matrices in the equation above, are expanded versions of $K^e$, $K_z^e$, $M^e$ and $M_z^e$. They are all $[10x10]$ versions of the original $[4x4]$ and $[2x2]$ matrices.

$$K_x^e = \begin{bmatrix} K_a^e & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_c^e & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (3.51)
Note that the $K^e_a$, $K^e_b$, $K^e_c$, $K^e_d$ and the homologous for $M^e$, are [2x2] submatrices, that were defined in eqs. 3.36 of section 3.2.2.1. The $K^e$ matrix resulting of equation 3.49 will be,
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\[ K^e = \begin{bmatrix}
K^e_a & 0 & 0 & K^e_b & 0 & 0 \\
0 & 0 & K^e_c & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & K^e_d & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

(3.54)

It is convenient again, to split this \( K^e \) and \( M^e \) into four submatrices to ease the assembly process that will lead to the global system, as described in the following section. Now we define,

\[ K^e = \begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\
K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\
K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \\
\end{bmatrix}, \quad \quad K^a = \begin{bmatrix}
K_{61} & K_{62} & K_{63} & K_{64} & K_{65} \\
K_{71} & K_{72} & K_{73} & K_{74} & K_{75} \\
K_{81} & K_{82} & K_{83} & K_{84} & K_{85} \\
K_{91} & K_{92} & K_{93} & K_{94} & K_{95} \\
K_{101} & K_{102} & K_{103} & K_{104} & K_{105} \\
\end{bmatrix}, \quad \quad K^b = \begin{bmatrix}
K_{16} & K_{17} & K_{18} & K_{19} & K_{110} \\
K_{26} & K_{27} & K_{28} & K_{29} & K_{210} \\
K_{36} & K_{37} & K_{38} & K_{39} & K_{310} \\
K_{46} & K_{47} & K_{48} & K_{49} & K_{410} \\
K_{56} & K_{57} & K_{58} & K_{59} & K_{510} \\
\end{bmatrix} \]

(3.55)

\[ K^c = \begin{bmatrix}
K_{66} & K_{67} & K_{68} & K_{69} & K_{610} \\
K_{76} & K_{77} & K_{78} & K_{79} & K_{710} \\
K_{86} & K_{87} & K_{88} & K_{89} & K_{810} \\
K_{96} & K_{97} & K_{98} & K_{99} & K_{910} \\
K_{106} & K_{107} & K_{108} & K_{109} & K_{1010} \\
\end{bmatrix} \]

(3.56)

The procedure to obtain \( M^e \) is exactly the same. The mass matrix will be exactly the same for all the finite elements. The stiffness matrix will vary at each element due to the variable tension along the length of the riser model.

3.2.3 Assembly of the equations

Once the solution is approximated in a generic element \( \Omega^e \), we have to extrapolate the solution to the whole mesh in order to obtain the assembled system that gives the approximate solution for the complete model. The assembly of the equations is based on two concepts:
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1. Continuity of the primary variables.

2. Equilibrium of the secondary variables.

The first concept is directly solved changing from local notation to global notation, this means the values of the variables at the second node of element $\Omega^{n-1}$ must be equal to the value of the variables at the first node of the $\Omega^n$ element. Then if the local notation for the the second node of element $\Omega^{n-1}$ was $z_{2}^{n-1}$ it now becomes $z_n$, the same for the first node of $\Omega^n$ that was before $z_1^n$. That is directly expressed in the connectivity matrix $B$ of the system. This matrix represents local nodes in columns, finite elements in rows and its components are the number of the global nodes. We can see that local node 2 of element $\Omega^1$ (row 1, column 2) is global node 2, local node 1 of element $\Omega^2$ (row 2, column 1) is global node 2, local node 2 of element $\Omega^2$ (row 2, column 2) is global node 3, and so on.

$$B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ \vdots & \vdots \\ \vdots & \vdots \\ n-2 & n-1 \\ n-1 & n \end{bmatrix}$$

Global node numbers appearing in more than one row indicate the contribution of one node to several finite elements. That is why the diagonal of the assembled matrix is formed by the addition of two components, except for global node 1 and n, which are the boundaries.

The second concept is based on the idea that the summation of the values of the secondary variables coming from adjacent elements at one node must represent equilibrium. Looking to the connectivity matrix we can construct,
K = 
\[
\begin{bmatrix}
K_1^1 & K_2^1 & 0 & \ldots & \ldots & \ldots & 0 \\
K_1^1 & (K_1^1 + K_2^1) & K_2^2 & 0 & \ldots & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & \ldots & K_c^e (K_d^e + K_n^e) & K_{b_1}^e + 1 & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & 0 & K_c^n (K_d^n + K_n^n) & K_b^n \\
0 & \ldots & \ldots & \ldots & 0 & K_c^n & K_d^n
\end{bmatrix}
\] (3.58)

which is the final assembled system, where superscript \(n\) denotes the total number of nodes in the riser. This scheme is valid for \(K\) and \(M\), and allows us to have the global system of equations that give the approximate solution of equations 3.4 and 3.9 for the riser model.

\[
Kr + M\ddot{r} = F_Q + Q = F
\] (3.59)

\(Q\) is the vector of secondary variables \((A.5)\), and it is previously known. In all the nodes inside the domain, \(Q_2^e + Q_1^{e+1} = 0\).
where $T_b$ and $T_t$ are the tensions at the bottom and top end of the riser. The reader must notice at this point that one of the goals of applying this method to the riser model used, is to find the load distribution along the axis of the riser. The input data in this problem will not be the loads but the response. By inputting into equation (3.59) it is possible to calculate the nodal forces and hence the transverse (drag and lift) fluid load distributions along the axis of the riser, see chapter 6.

### 3.3 Generalized eigenvalue problem

When solving time dependent problems using the method of separation of variables, eigenvalue problem formulations are obtained. Practically, all partial differential equations have an eigenvalue problem associated with them. In structural mechanics the eigenvalue problems are associated with the natural frequencies and the modes of vibration of the structures. The eigenvalue problems and their properties are described by the Sturm-Liouville theory, ([Masdemont-Soler 2002](#)).

The separation of variables method is based on finding a solution for an equation of the form $u(z,t) = p(z)q(t)$, so the object function is being decomposed into two, that depend only on one variable. Applying this concept to the transverse motion model equation

$$\frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 u(z,t)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z,t)}{\partial z} \right) + m \frac{\partial^2 u(z,t)}{\partial t^2} = 0$$

$$q(t) \left[ \frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 p(z)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial p(z)}{\partial z} \right) \right] + p(z)m \frac{\partial^2 q(t)}{\partial t^2} = 0$$

Dividing now the equation by $u(z,t) = p(z)q(t)$ and by $m$ and separating the functions depending on the same variable at each side of the equation one obtains

$$\frac{1}{mp(z)} \left[ \frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 p(z)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial p(z)}{\partial z} \right) \right] = -\frac{1}{q(t)} \left( \frac{\partial^2 q(t)}{\partial t^2} \right)$$
The left hand side of the equation only depends on \( z \), and the right on \( t \). Note that if both sides are considered separately, both results should be equal and constant because they are scalars. If this constant value is defined as \( \lambda \), the eigenvalue problem is obtained,

\[
\frac{1}{mp(z)} \left[ \frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 p(z)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial p(z)}{\partial z} \right) \right] = -\frac{1}{q(t)} \left( \frac{\partial^2 q(t)}{\partial t^2} \right) = \lambda
\]

Eigenvalue systems are considered to be the spatial side of the equation, then

\[
\frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 p(z)}{\partial z^2} \right) - \frac{\partial}{\partial z} \left( T(z) \frac{\partial p(z)}{\partial z} \right) - \lambda mp(z) = 0 \quad (3.61)
\]

where the eigenvalues are the values of \( \lambda \) that produce a non trivial solution for the equation with its boundary conditions applied. In structural mechanics, eigenvalues are associated with the natural frequencies of vibration. Eigenvectors are the result of substituting each eigenvalue in the equation, and are associated with the modal shapes of the vibration, \( \Phi \). Applying the finite element method using the same procedure as explained in section \( \Lambda \), first obtaining the weak formulation of (3.61), an algebraic system is obtained, the generalized eigenproblem. Note that that the generic function \( p(z) \) has been substituted by the mode shapes matrix \( \Phi \) in the algebraic system, where each column vector is the \( i^{th} \) mode shape \( \phi_i \).

\[
K\Phi - \Lambda M\Phi = 0
\]

\[
[K - \Lambda M] \Phi = 0 \quad (3.62)
\]

\( K \) and \( M \) are the stiffness and consistent mass matrices for the global system and \( \Lambda \) is a diagonal matrix with components \( \lambda_{ij} = \omega_i^2 \) (\( \forall i = j \)), being \( \omega_i \) the natural frequencies of the system, expressed in \( \text{rad/s} \). Stiffness and mass matrix are symmetrical and all their values are real, so real eigenvalues and eigenvectors are expected. The \( K \) and \( M \)
matrices can be obtained by applying the assembly procedure explained in section 3.2.3, leading to the assembled matrices of one of the transverse equations, which are then used in equation 3.62. There are several ways of solving eigenvalue problems: power method, direct method and QR and QZ methods are the most often used ones. There are several mathematical packages and mathematical libraries that allow calculation directly introducing the right matrices. The software package used for this purpose uses the QZ method, see the MATLAB manual, (Mathworks 2000).

The solution of the generalized eigenproblem in equation 3.62, yields the \( n \) eigen-solutions \((\lambda_1, \phi_1), (\lambda_2, \phi_2), \ldots, (\lambda_n, \phi_n)\). One of the properties of the eigenvectors is that they are M-Orthonormal, i.e.,

\[
\phi_i M \phi_j \begin{cases} = 1, & i = j \\ = 0, & i \neq j \end{cases}
\]  

or

\[
\Phi^T M \Phi = I
\]  

(3.64)

and since the eigenvectors are M-Orthonormal,

\[
\Phi^T K \Phi = \Lambda
\]  

(3.65)

A comparison between the natural frequencies obtained through this method against the measured natural frequencies is presented in tables 5.1 and 5.2 of section 5.

### 3.4 Structural Damping

The structural damping can be included into the system using the proportional or Rayleigh model for damping.
\[ \mathbf{M} \ddot{\mathbf{r}} + \mathbf{C} \dot{\mathbf{r}} + \mathbf{K} \mathbf{r} = \mathbf{F} \]  

(3.66)

where \( \mathbf{K} \) is the stiffness matrix, \( \mathbf{C} \) the damping matrix, \( \mathbf{M} \) the mass matrix and \( \mathbf{r} \) the displacement vector. The dot indicates differentiation with respect to time. The proportional model assumes that \( \mathbf{C} \) is C-Orthonormal,

\[ \phi_i^T \mathbf{C} \phi_j = 2\omega_n \xi_i \delta_{ij} \]  

(3.67)

where the \( \delta_{ij} \) is the Kronecker delta, defined as,

\[ \delta_{ij} \begin{cases} 
1, & i = j \\
0, & i \neq j 
\end{cases} \]  

(3.68)

hence \( \mathbf{C} \) can be defined as the linear combination of the stiffness and mass matrices,

\[ \mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \]  

(3.69)

Coefficients \( \alpha \) and \( \beta \) can be calculated using modal structural damping ratios and the natural frequency of the corresponding modes, see section 5.3 in chapter 5. Structural damping ratios were obtained from decay tests in air, assuming that the air damping could be neglected. Substituting equation 3.69 into equation 3.67 and using equations 3.64 and 3.65,

\[ \alpha + \beta \omega_i^2 = 2\omega_i \xi_i \]  

(3.70)

which gives the damping ratio as a function of the natural frequency.
Chapter 4

Experimental Set-up

In this chapter details are given of the experimental set up used to investigate the behaviour of the riser model. The main objective of these experiments is to reproduce at small scale the excitation and the response of deep water risers subjected to stepped uniform currents. Experiments were carried out at the Delft Hydraulics Water Laboratories facilities. The company has a large facility laboratory in the North East Polder (The Netherlands), where the Delta Flume is located. The Delta Flume consists of a 240 m length flume with a cross section of 5 m wide by 7 m deep, equipped with a towing carriage able to deliver speeds up to 1 m/s. The diameter of the model riser was 28 mm and the Reynolds number ranged between 2800 and 28000. The experiments where carried out during five weeks, starting in May 2003.

4.1 Experiment description

The riser model was installed on the carriage as given in the layout shown in figure 4.1. The lower 45% of its length was subjected to a uniform current, the rest was in still water. This was possible because the riser passed inside a long rectangular tank sealed at the top and open at the bottom. A pump connected to the sealed end was used to empty the air inside, filling the tank with water. The exact submerged length, that
is the distance from the bottom of the riser to the bottom of the vacuum tank, was 5.94m. The tank was supported by an A-frame structure fixed to the carriage as seen in figure 4.1. Underneath the water surface, hanging from the lower part of the carriage, a scaffolding structure was constructed in order to provide a fixing for the bottom of the riser. A pair of universal joints connected between them by a steel rod, was used at each end of the riser model, to attach it to the supporting structure restricting torsion. The distance between the bottom of the riser and the floor of the flume was approximately 5cm. This set-up modeled an idealised off-shore situation where a riser is partially subjected to a zero current over half its length and to a uniform current profile over the rest. The speed of the carriage was varied from one run to another but always inside the range of 0.1 m/s to 1 m/s.

Figure 4.1: Layout of the experiment
4.1.1 The Delta Flume facility

The Delta Flume facility is located in the Noth-East Polder in the Netherlands, and it is owned by Delft Hydraulics Water Laboratories. It consists of a 240 m long water channel, with a cross section of 5x7 m. The maximum water depth in the flume for the experiments was around 6 m. The channel can be seen in figure 4.2. It also has a towing carriage able to deliver velocities up to 1 m/s by moving over two rails situated along the length of the channel.

![Image of Delta Flume facility](image)

Figure 4.2: The Delta Flume at Delft Hydraulics lab

4.1.2 The riser model

The riser model was designed by a team lead by Prof. J.R. Chaplin at Southampton university. To obtain the desired structural parameters, it was made by machining a number of Phosphor bronze rods, each consisting of an 8 mm diameter core with
diaphragms of 25 mm diameter along the length. These diaphragms were spaced at 9 mm, and had a thickness of 3 mm, they also had cut outs to allow the instrumentation cables to pass along the model. A plastic tube of 28 mm diameter was used to cover the bronze structure. The cover was made of fluoroplastic (FEP) and had a very smooth surface. Figures 4.3 and 4.4 show the riser in detail. The flexural stiffness of the riser $EI$, was 29.88 Nm$^2$, where $E$ is the Young’s modulus and $I$ is the section inertia. A tensioning system was installed and used to change the tension in the riser between 380 N and 1900 N, therefore, the submerged fundamental natural frequency of the riser varied from 0.4 Hz to 1.1 Hz, depending on the applied tension. The riser was formed by the connection of sixteen sections, each of length 820 mm, giving a total length of 13.12 m.

Figure 4.3: Riser detail: Strain gauge location and cross sectional view

The mass of the riser (1.279 kg/m), taking into account the instrumentation cables (0.094 kg/m), the plastic cover (0.092 kg/m) and the internal water once submerged (0.384 kg/m) was 1.848 kg/m. The mass ratio $m^*$, was 3,

$$m^* = \frac{m}{\rho \frac{\pi D^2}{4}} ,$$

where $m$ is the mass per unit length of the riser, $\rho$ is the water density and $D$ is the external diameter of the model.

The last set of tests, carried out during the last testing day, was performed to analyze the response of the riser with bumps as a vortex shedding suppression device.
bumps were manufactured based on the design recommendations by Owen (2001). A
good performance of the bumps was noticed in previous work at the Imperial College
water laboratory, in rigid cylinder response experiments, achieving a reduction of drag
and displacements by disrupting the vortex shedding. For more details one can read
the PhD theses of Owen (2001) and Brankovic (2004).

In the part subjected to the current, 234 bumps were used. They were distributed
in diametrically opposite pairs every 50 mm along the length. The orientation was
varied 45 degrees from one pair to the other, producing a longitudinal helical shape. A
photograph of the riser with the bumps stuck on the surface can be seen in figure 4.5.
The total mass of the bumps was 0.7432 gr, producing an increase of the mass per unit
length in the lower part of the riser of 0.121 kg/m. The submerged weight changed
as well, not only because the change in mass but also because of the buoyancy was
affected by adding the bumps. The average volume of one bump is 2.857 ml, producing
an overall increase of 6.685 × 10^{-4} m^3. The submerged weight of the bumpy riser was
found to be slightly higher to that of the plain riser, with a value of 12.25 N/m. Table 4.1 summarizes the mass characteristics of the riser, presenting its mass and weights under the different test situations. By combining the values presented in table 4.1 one can model the riser and calculate more accurately the tension variation along the length of the riser and therefore obtain better mass and stiffness matrices when solving eigenproblems.

![Figure 4.5: Detail of the riser model with the suppression bumps attached to its surface](image)

### 4.1.3 Instrumentation

Thirty-two pairs of strain gauges were distributed along the riser, and attached to the internal core, with the pairs spaced 410 mm and oriented roughly in the in-line $x$ and in the cross-flow $y$ directions. The first pair was placed at 205 mm from the bottom, and the last one 205 mm from the top. Figure 4.3 shows a detail of the strain gauge installation on the riser. The riser model also included two waterproof accelerometers at the middle of its length and one more at a quarter length from the bottom. Four more accelerometers were installed: two of them measuring in $x$ and $y$ directions near the bottom of the riser in the scaffolding structure, and the other
two measuring also in $x$ and $y$ directions, were placed at the top of the rectangular tank. These accelerometers monitored the vibration of the whole structure to ensure it was not vibrating at the natural frequency of the supporting structure, which was distant enough to that expected for the riser model. Four flow meters were installed at different heights of the scaffolding structure to certify that the riser was subjected to a uniform current profile when there was a relative flow.

Figure 4.6: External accelerometers at the top of the tank and bottom of the structure

In addition, two load cells were used to measure the tension in the riser and two more to measure the drag. The tension load cells were installed between the universal joints and the ends of the riser. The drag load cells were installed at the top, as well as at
the bottom, using a rod parallel to the direction of the flow, connecting the load cells to the end of the riser. Details are shown in figure 4.8.

Figure 4.8: Bottom tension and drag load cells

A mechanical excitation device connected to the bottom of the riser with steel cables was used to determine the mechanical behaviour of the riser both in air and in still water. By driving the riser at its bottom end with this device, natural frequencies and damping coefficients were determined. A LVDT displacement sensor was attached to the bottom of the riser to check the displacements at this point. They can be seen in figure 4.9.

Figure 4.9: Bottom LVDT sensor and oscillating device

At the top of the rectangular tank a system was placed that allowed us to change the tension applied to the riser, figure 4.10. This system was based on several springs connected between two u-shaped metallic bars. The lower bar was connected directly to the riser, and the upper one was supported at its ends by two long bolts. When
these bolts were turned the upper bar moved up or down, increasing or decreasing respectively the tension in the riser through the spring system.

Figure 4.10: Tension system

The bottom end of the riser was connected to a plate in the scaffolding structure through the universal joints and the steel rod, see detail in figure 4.11. In the upper part of the riser, inside the rectangular cross-section tank, the riser was attached to the u-shaped bar of the tensioning system by means of the other pair of universal joints and rod.

Figure 4.11: Bottom universal joint and Hammamatsu camera

Once everything was installed and before the first acquisition of data, a digital camera was used to obtain directly the displacements of the riser at a point where a pair of strain gauges was located. A target was fixed besides the strain gauges, approximately
at mid length of the riser. Later, the displacements acquired with the camera were compared with the ones calculated with the strain gauges. With this camera and the external accelerometers, three independent measurements were available at the same point to assess the accuracy of the displacements calculated from the strain gauge outputs. Tables 4.1 and 4.2 summarize the main characteristics of the experimental set up.

<table>
<thead>
<tr>
<th>RISER MASS CHARACTERISTICS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PLAIN</strong></td>
</tr>
<tr>
<td>mass</td>
</tr>
<tr>
<td>(kg/m)</td>
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<tr>
<td>Full Tank</td>
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<tr>
<td>Empty Tank</td>
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Table 4.1: Riser mass characteristics

<table>
<thead>
<tr>
<th>EXPERIMENT MAIN PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>External diameter</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Aspect ratio</td>
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<td>Submerged Length</td>
</tr>
<tr>
<td>Flexural Stiffness</td>
</tr>
<tr>
<td>Top Tension</td>
</tr>
<tr>
<td>Flow speeds</td>
</tr>
<tr>
<td>Mass ratio</td>
</tr>
<tr>
<td>Mass ratio (bumpy cylinder)</td>
</tr>
<tr>
<td>Reynolds number</td>
</tr>
<tr>
<td>Fundamental Natural Frequency</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of main parameters of the experiment
4.2 Experiments

More than 185 runs were performed producing an extensive set of data. The items below show a general classification of all the tests performed. The reader must note that the top tension was set before the beginning of each run to the desired value, meaning that for each carriage speed tested top tension changed as hydrodynamic loads were applied. The main tests were as follows:

- Decay tests in air/water to find damping coefficients
- Ramped frequency tests in air/water to find natural frequencies
- Constant carriage speed runs
  - Upper cylinder full of air
  - Sealed cylinder full of water
Chapter 5

Experimental Results

In this chapter, results for all the tests carried out at the Delta Flume are presented. Part of the results shown in this chapter, were presented at the International Conference on Flow-Induced Vibrations 2004 (Chaplin et al. 2004) and in the Journal of Fluids and Structures (Chaplin et al. 2005).

The first section explains how the data has been analyzed by treating one test case as an example. A discussion about the mechanical properties of the riser is presented in the second and third sections, showing the vibrational behaviour of the riser. The rest of the chapter shows the results for the plain cylinder and a comparison with the bumpy model. Runs are divided into sets of tests according to the different top tensions used. In some cases the upper tank was empty of water and these cases are shown as well. The analysis is shown as a result of using sinusoidal mode shapes because the results obtained by using FEM calculated mode shapes were practically the same. Finally, the results of several runs are compared against results obtained with one of the most extensively used prediction codes, SHEAR7.
5.1 Data Analysis and Experimental Uncertainty

The data obtained was sampled at a rate of 200 Hz, giving a Nyquist frequency of 100 Hz. This frequency was determined to be high enough to avoid aliasing problems, because the expected response frequencies of the riser were much lower than this. Three acquisition cards were used for the following purposes:

▷ The first of the three acquisition boards digitized the 32 strain gauges oriented in the \( x \) line direction, which is parallel to the length of the flume.

▷ The second one was dedicated to the other 32 strain gauges placed at the same points along the riser axis, but oriented in the cross-flow \( y \) direction.

▷ The third board acquired all the rest of the instrumentation needed for the experiment: the internal and external accelerometers, the load cells, the LVDT displacement sensor, the carriage speed and the flow meters.

Applying the calibration factors to the channels, the raw data was transformed into physical units in the metric system, strain gauge signals were then processed to give local riser curvatures. A discrete function of the curvature of the riser as a function of its length and time was obtained. The curvature was measured at 32 equally spaced points along the axis of the riser as specified in the previous chapter. For analysis purposes, the curvatures were assumed to vary linearly between the measurement points and they were linearly interpolated to produce 128 nodes. A cartesian reference frame in which \( x \) corresponds to the relative flow direction, \( y \) corresponds to the normal direction and \( z \) to the vertical direction, has been considered. The curvature \((c_x, c_y)\) of the riser can be expressed as follows:

\[
\begin{align*}
    c_x(z, t) &= \frac{\partial^2 u(z, t)}{\partial z^2} \\
    c_y(z, t) &= \frac{\partial^2 v(z, t)}{\partial z^2}
\end{align*}
\] (5.1)
where $u$ are $v$ are the in-line and transverse motions. Displacements are the solution of the second integration of the curvatures along the direction of the axis of the riser:

$$u(z, t) = \int_0^L \int_0^L \frac{\partial^2 u(z, t)}{\partial z^2} dz \, dz$$

$$v(z, t) = \int_0^L \int_0^L \frac{\partial^2 v(z, t)}{\partial z^2} dz \, dz$$

being $L$ the total length of the riser model. As already mentioned in the previous chapter, the riser was linked to the supporting structure and to the tank with universal joints at its ends. No torsional movements were allowed, rotation was only permitted around the two axes of the universal joints. These axes were in the $xy$ plane, perpendicular to the riser axis which was parallel to $z$. The boundary conditions can be expressed in the same way as for a simply supported or pin ended beam, that is no displacements and no curvatures at the ends.

$$u(0, t) = v(0, t) = 0, \quad u(L, t) = v(L, t) = 0 \quad \forall t$$

$$\frac{\partial^2 u(0, t)}{\partial z^2} = \frac{\partial^2 v(0, t)}{\partial z^2} = 0, \quad \frac{\partial^2 u(L, t)}{\partial z^2} = \frac{\partial^2 v(L, t)}{\partial z^2} = 0 \quad \forall t$$

An important objective of the experiments, was to obtain modal information from the data. A better understanding of the response of the riser can be achieved by analyzing the contribution of the different modes participating in the global response. The starting point for modal analysis is an assumption: the modal shapes. By assuming the modal shapes, the modal contributions can be found, therefore, the choice of the modal shapes is very important. Beam theory predicts sinusoidal mode shapes when no tension is applied to the beam. It predicts non-sinusoidal mode shapes depending on the tension applied to the beam. Because the finite element method (FEM) has been used to calculate eigenvalues and eigenvectors as detailed in section 3.3, they will be referred from now on as FEM mode shapes. The choice of the mode shapes will define the modal contributions, and they will be different for each choice, hence it is a source of uncertainty.
Modal analysis is based on the fact that the riser response can be expressed in matrix form as a linear combination of its modes. In the following paragraphs the in-line case is used to specify the equations, but substituting $U$ per $V$ and $A_x$ per $A_y$, they are valid for the cross-flow case.

$$U(z, t) = \Phi(z)A_x(t) \quad (5.4)$$

where $\Phi = [\phi_1, \phi_2, ..., \phi_n]$ is the displacement modal shapes matrix and its column components are the non-dimensional displacement mode shapes normalized to be 1 at its maximum. $U_x(z, t)$ is the nodal displacements matrix, with the time series of the displacements being its row vectors, $U_x(z, t) = [u_1(t), u_2(t), ..., u_n(t)]$. $A_x(t)$ is the instantaneous displacement modal amplitudes matrix, and it has the same units as $U(z, t)$, because the mode shapes are non-dimensional. Its row vectors are the time series of the displacement modal amplitudes at different heights along the length of the riser model, $A_x(t) = [A_{x1}(t), A_{x2}(t), ..., A_{xn}(t)]$. Bold letters are used to indicate matrices or vectors. For the in-line case, the static deformation or mean deflected shape is removed, and it is only used the dynamic part of the deflections. The time history of each displacement modal amplitude can then be obtained, as:

$$A_x(t) = \Phi^{-1}(z)U(z, t) \quad (5.5)$$

$$A_y(t) = \Phi^{-1}(z)V(z, t) \quad (5.6)$$

The same concept can be applied to the nodal curvatures matrix, $C_{x,y}(z, t)$, with its row vectors being the time series of the curvatures at different $z$ positions, $C_{x,y}(z, t) = [c_{x,y1}(t), c_{x,y2}(t), ..., c_{x,y_n}(t)]$. The subscripts $x,y$ indicate in-line and cross-flow respectively. The curvatures are the spatial second derivatives of the displacements, see equation [5.1]

$$C_x(z, t) = \frac{\partial^2 U(z)}{\partial z^2} = \frac{\partial^2 \Phi(z)}{\partial z^2} A_x(t)$$
\[ C_y(z, t) = \frac{\partial^2 V(z)}{\partial z^2} = \frac{\partial^2 \Phi(z)}{\partial z^2} A_y(t) \]

\[ C_{x,y}(z, t) = \Phi_c(z) A_{x,y}(t) \quad (5.7) \]

\[ A_{x,y}(t) = \Phi_c^{-1}(z) C_{x,y}(z, t) \quad (5.8) \]

where

\[ \Phi_c(z) = \frac{\partial^2 \Phi(z)}{\partial z^2} \quad (5.9) \]

and \( \Phi_c = [\phi_{c1}, \phi_{c2}, ..., \phi_{cn}] \) is the curvature modal shapes matrix and its column components are the non-dimensional curvature mode shapes. If \( \Phi_c(z) \) is normalised to have a value of 1 at the maximum of each column vector, \( \Phi_{cn}(z) \) is obtained, and the curvatures can then be expressed as:

\[ C_{x,y}(z, t) = \Phi_{cn}(z) A_{c_{x,y}}(t) \quad (5.10) \]

and the curvature modal amplitudes \( A_{c_{x,y}}(t) \), in the same units as \( C_{x,y}(z, t) \), are:

\[ A_{c_{x,y}}(t) = \Phi_{cn}^{-1}(z) C_{x,y}(z, t) \quad (5.11) \]

Looking to these equations, it can be noticed that another source of uncertainty could be produced by the instrumentation. It is due to the fact that the curvature is proportional to the square of the mode number. The expression of the sinusoidal mode shapes and its second derivatives is

\[ \phi_n(z) = \sin\left(\frac{n\pi z}{L}\right) \quad (5.12) \]
When the riser is vibrating at higher modes, the curvature associated with the lower modes is very small compared to that of the higher ones. From a practical point of view, when measuring strain, in cases in which the riser was vibrating at high modes, curvatures of the lower modes might be in the level of the electrical noise. In a first data analysis stage, the modal contributions from the raw data were calculated, observing that even in cases where the response was characterized by high modes, the fundamental mode practically always appeared. Figures 5.1 and 5.2 show the first 8 displacement modal amplitudes ($A_{x,y_i}(t)$, $i = 1, 2, ..., 8$) in a test case in which the response was mainly dominated by the 7th mode in-line and the 4th cross-flow. The spectra are also shown on the right hand side of the figures, showing an in-line dominant frequency of 7.04 Hz and a transverse frequency of 3.52 Hz. The flow speed was 0.61 m/s with mean top tension of 1610 N. The values in both figures are non-dimensional, with the axis ranging from -1 to 1 diameter cross-flow, and from -0.5 to 0.5 diameters in-line. A very high contribution corresponding to the first mode is observed, both in-line and cross-flow. The standard deviations ($\sigma_{A_{xy}}$) of the curvature modal amplitudes ($A_{c_i}(t)$, $i = 1, 2, ..., 32$) associated with the $i$th mode, are shown in the upper plots of figure 5.3. The lower plots in the same figure shows the standard deviation of the curvature associated with modal amplitudes of modes 1 to $n$. This is in fact, a measure of the energy content of the curvature signals. The curvature associated with the first mode is very small compared to that of the dominant ones. For the test case depicted, it is only 1.71% of the one associated with the dominant mode, even though figures 5.1 and 5.2 show very important modal weight for the first mode. Because of this problem, a method based on the identification of the curvature associated to each mode participating in the vibration, was adopted to discriminate the lower experimental spurious modes. Equation 5.8 was used to obtain the modal contributions from the raw measured curvatures. The fundamental mode was excluded, when the standard deviation of its associated curvature was less than 3% of the summation of the standard deviation of the curvatures of all calculated modes. This threshold can be seen as a dashed line in figure 5.3 in which the first mode should be neglected according to this
Due to the same problem, it was observed that the in-line direction curvatures required in some cases, the correction of not only the fundamental mode around the mean or static deflection, but also up to the 2nd, 3rd or 4th, according to the best agreement with the external accelerometer measurements.

Another practical issue with the data appeared when attempting the calculation of the fluid distributed load along the axis of the riser model, see chapter 6. As it will be explained, the experimental data was used as the input to a structural numerical model of the riser, in order to obtain the load distribution along the axis of the riser. The first attempts were unsuccessful because of the apparent noise in the input signals. Initially, the displacement data seemed not to be contaminated, but when inputed into the FEM model, the results were extremely noisy. By using 32 strain gauges in each measuring...
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Figure 5.2: Example: Non-dimensional cross-flow displacement modal amplitudes from raw data - $\bar{T}_t = 1610N \cdot 0.61 \text{ m/s}$
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Figure 5.3: Example: Standard deviation of mode n and cumulative standard deviation of curvatures of modes 1 to n - $\bar{T}_i = 1610N - 0.61m/s$
direction, the highest possible mode identifiable by sampling at equally spaced positions along the axis of the riser, is the 32\textsuperscript{nd}. Following this statement, the first question that comes out is: Why should one be dealing with modes higher than the 32\textsuperscript{nd}? The answer is that when using the modal decomposition technique described by equation \ref{eq:mod}, the number of mode shapes used has to be the same as the number of points in which the riser is discretised. So if 128 nodes are used, 128 modal shapes are needed, but obviously mode shapes higher than the 32\textsuperscript{nd} should have zero modal contribution because physically they do not exist. Because the instantaneous response of the riser pipe is not exactly as theoretical (sinusoidal or FEM) mode shapes represent, leakage exist in a form of the signal being distributed into non real modes, those higher than the 32\textsuperscript{nd}. It was observed that the energy contained by the modes above the 32\textsuperscript{nd}, was in the worst case 3\% of the signal’s total energy. By low pass filtering the signals with a Finite Impulse Response filter and setting to zero modes higher than the 32\textsuperscript{nd}, the overall response was not modified, but the result was a signal with very low noise levels, that could then be used as the input to the structural FEM model. The mode elimination described here has been done by setting $\phi_i$ to zero in $\Phi_c$ when necessary. The same applies to the filtering, which has been done individually for each $\phi_i$.

Once $\Phi_c$ was corrected as explained in the above paragraphs, equation \ref{eq:curv} was used to obtain the new curvatures. After these modifications, the displacements were calculated as in equation \ref{eq:dis}. The result of this numerical integration was compared then with the displacements measured by means of the accelerometers installed in the riser, and with the digital camera measurements. The agreement between these independent measurement techniques was very good after the corrections.

\section*{5.2 Natural Frequencies and Mode Shapes}

Experimental natural frequencies calculated by analyzing the ramped frequency tests are shown in this section. Comparisons with FEM calculated natural frequencies are presented as well. In order to find the experimental natural frequencies, the riser was
Figure 5.4: Example: Non-dimensional cross-flow displacement modal amplitudes of a ramped frequency test (May19 C001) - $T_i = 1188N$

excited at its bottom before each set of tests, after setting each different top tension. An excitation device introduced a pure cross-flow alternating displacement of fixed amplitude with increasing frequency. The excitation frequency was ramped up to 16Hz resulting in a response characterized by almost pure modes. Figure 5.4 shows the modal amplitudes of the first ten modes of one of the ramped frequency tests and figure 5.5 shows the spectra of these signals in which the peaks indicate the natural frequencies of the riser.

Tables 5.1 and 5.2 show the first ten experimental and FEM natural frequencies for the same tensions.

The results presented in table 5.2 have been produced solving the generalized eigenvalue problem as described in section 3.3, using 65 nodes. The added mass was computed by calculating the mass of displaced water per unit length as follows,

$$m_{\text{added}} = \rho \frac{\pi}{4} D^2$$

(5.13)
Figure 5.5: Example: Spectra of the non-dimensional cross-flow displacement modal amplitudes of a ramped frequency test (May 19 C001) - $T = 1188 N$

### Table 5.1: First 10 experimental natural frequencies

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean $\bar{T}$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
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<th>$f_{10}$</th>
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<tbody>
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<td>397.22</td>
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<td>0.85006</td>
<td>1.2801</td>
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<td>4.5936</td>
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<td>6.0571</td>
<td>6.7438</td>
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<td>0.9534</td>
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<td>8.3952</td>
</tr>
<tr>
<td>E001 May26</td>
<td>1919.40</td>
<td>1.11</td>
<td>2.2544</td>
<td>3.3164</td>
<td>4.526</td>
<td>5.5832</td>
<td>5.4808</td>
<td>5.4632</td>
<td>5.4043</td>
<td>11.675</td>
<td>10.378</td>
</tr>
</tbody>
</table>

Equation 5.13 is valid for circular cylinders at low Keulegan-Carpenter ($KC = \frac{V T}{D}$) and $\beta = \frac{Re}{KC}$ parameters (see Sumer & Fredsø (1997) and Sarpkaya & Isaacson (1981)).

In tables 4.2 and 4.1 of section 4.1, the main properties of the riser have been presented. The linear mass of riser submerged in water plus its cover and instrumentation cables is 1.845 kg/m. The added mass calculated as in equation 5.13 is 0.616 kg/m, then the mass per unit length of the riser taking into account the effect of the added mass,
for the case of the riser submerged in water, is $m=1.845+0.616=2.461$ kg/m. In cases where the riser was surrounded by air, the effect of the added mass compared to the mass of the model was very small and it can be neglected. The agreement between the different methods to calculate the natural frequencies is clear from the results, and it can be concluded that the FEM code is accurate.

Sinusoidal and FEM mode shapes are compared in the following figures. The first 6 FEM mode shapes for two different tensions are presented in figure [5.6] versus sinusoidal ones. Although FEM mode shapes are a better mathematical approximation for problems with varying tension along the length, the figure shows how sinusoidal mode shapes and high tension FEM mode shapes are practically identical. Therefore, for low tension cases, FEM mode shapes are a better assumption, whilst for the high tension cases, the difference between FEM and sinusoidal is not appreciable.

Figure [5.7] shows one of the tests where the riser was forced to respond in almost pure cross-flow modes (ramped frequency tests). FEM mode shapes and the experimental response are compared by normalizing the response to be 1 at its maximum. FEM mode shapes show better agreement than sinusoidal ones, and they clearly show the effect of the tension that produces higher deflections in the lower part of the riser.

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean $\bar{f}$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
<th>$f_9$</th>
<th>$f_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A000</td>
<td>May21</td>
<td>397.22</td>
<td>0.42349</td>
<td>0.85503</td>
<td>1.7685</td>
<td>2.2631</td>
<td>2.7903</td>
<td>3.3546</td>
<td>3.9602</td>
<td>4.6105</td>
<td>5.3085</td>
</tr>
<tr>
<td>B000</td>
<td>May21</td>
<td>428.00</td>
<td>0.44301</td>
<td>0.89361</td>
<td>1.8429</td>
<td>2.3538</td>
<td>2.8961</td>
<td>3.4744</td>
<td>4.0928</td>
<td>4.7546</td>
<td>5.4631</td>
</tr>
<tr>
<td>B002</td>
<td>May16</td>
<td>812.96</td>
<td>0.65626</td>
<td>1.3173</td>
<td>1.9876</td>
<td>2.6716</td>
<td>3.3737</td>
<td>4.0979</td>
<td>4.8482</td>
<td>5.6281</td>
<td>6.4412</td>
</tr>
<tr>
<td>C001</td>
<td>May15</td>
<td>815.47</td>
<td>0.66</td>
<td>1.32</td>
<td>1.99</td>
<td>2.67</td>
<td>3.37</td>
<td>4.10</td>
<td>4.85</td>
<td>5.63</td>
<td>6.44</td>
</tr>
<tr>
<td>A002</td>
<td>May15</td>
<td>829.63</td>
<td>0.66214</td>
<td>1.329</td>
<td>2.0051</td>
<td>2.6947</td>
<td>3.4023</td>
<td>4.1318</td>
<td>4.8871</td>
<td>5.672</td>
<td>6.4896</td>
</tr>
<tr>
<td>A001</td>
<td>May15</td>
<td>830.68</td>
<td>0.6619</td>
<td>1.3286</td>
<td>2.0044</td>
<td>2.6938</td>
<td>3.4011</td>
<td>4.1304</td>
<td>4.8855</td>
<td>5.6701</td>
<td>6.4876</td>
</tr>
<tr>
<td>A001</td>
<td>May23</td>
<td>1050.10</td>
<td>0.75357</td>
<td>1.5113</td>
<td>2.277</td>
<td>3.0547</td>
<td>3.8482</td>
<td>4.6612</td>
<td>5.4972</td>
<td>6.3595</td>
<td>7.2514</td>
</tr>
<tr>
<td>C001</td>
<td>May19</td>
<td>1188.28</td>
<td>0.80865</td>
<td>1.6211</td>
<td>2.4411</td>
<td>3.2722</td>
<td>4.1181</td>
<td>4.9822</td>
<td>5.8679</td>
<td>6.7783</td>
<td>7.7166</td>
</tr>
<tr>
<td>B001</td>
<td>May20</td>
<td>1534.68</td>
<td>0.93</td>
<td>1.86</td>
<td>2.79</td>
<td>3.74</td>
<td>4.70</td>
<td>5.67</td>
<td>6.67</td>
<td>7.68</td>
<td>8.72</td>
</tr>
<tr>
<td>A004</td>
<td>May23</td>
<td>1676.40</td>
<td>0.9711</td>
<td>1.9454</td>
<td>2.9259</td>
<td>3.9157</td>
<td>4.9178</td>
<td>5.9353</td>
<td>6.9709</td>
<td>8.0275</td>
<td>9.1078</td>
</tr>
</tbody>
</table>

Table 5.2: First 10 FEM Natural Frequencies
Figure 5.6: Comparison between low and high tension FEM vs. sinusoidal mode shapes (normalised to be 1 at their maximums). Vertical axes show the elevation along the riser model in m.
Figure 5.7: Comparison of the experimental response in a ramped frequency test (May15 A001) against FEM mode shapes (normalised to be 1 at their maximums). Vertical axes show the elevation along the riser model in m.
5.3 Decay tests in air: Structural Damping

The vibrational behaviour of a flexible beam is characterized by its natural frequencies, mode shapes and modal damping ratios. The aim of this section is to find the structural damping characteristic of the riser model, by analyzing the acquired data. Decay tests in air were carried out under different experimental conditions. Neglecting the fluid damping due to the surrounding air, structural damping can be calculated. The riser was excited at its bottom end using the same excitation device as in the ramped frequency tests described previously. The model was driven up to the desired mode and once the vibration was consistent in a specific mode, the excitation source was stopped letting the model go to its natural equilibrium position. By analyzing the displacement modal amplitude decay process, and assuming that the vibration was occurring purely at the desired mode, the modal damping ratio can be calculated. The example shown below is a decay test in air carried out with a top tension of 887 N, driving the riser model to a vibration in its 6th cross-flow mode at a frequency of 5.38 Hz with an approximate initial amplitude of 0.35 diameters. Solid lines in figure 5.8 show the time series of the dominant modal amplitude. Peaks are shown as open circles and the envelopes, which consist of the exponential fit of the amplitude signal, as thick solid lines. The equation of each edged envelope (upper and lower) and the damping ratios of each one, as a percentage of the critical damping, are

\[ e(t)_{up}^6 = 0.358e^{-0.0455t} \quad \xi_{up}^6 = 0.365\% \]
\[ e(t)_{low}^6 = -0.353e^{-0.0130t} \quad \xi_{low}^6 = 0.385\% \]

where the subscript indicates the mode number. The mean damping ratio between the upper and lower envelopes, has been used as the final value for each test. In the case shown it results in:

\[ \xi_6 = 0.375\% \]

In a multi-modal problem such as this one, a different decay test should have been
Figure 5.8: Decay test in air
carried out for each experimental condition and for each mode. It would then have been possible to calculate modal damping ratios for each mode under each tension. Because it was observed that the model behaved in a similar way, only some of the experimental conditions were tested. Table 5.3 shows the damping ratios calculated from three decay tests in air, when the riser model was vibrating in three different conditions. Vandiver (1983) reported similar critical modal damping ratios in his experiments. He found the values to be within the range of 0.1 and 0.5% at a tension of 4450 N, and within 0.2 and 0.5% at 3523 N, when vibrating at the 2,3 and 4th mode.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency (Hz)</th>
<th>Frequency (rad/s)</th>
<th>ξ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.74</td>
<td>10.93</td>
<td>0.257</td>
</tr>
<tr>
<td>4</td>
<td>3.51</td>
<td>22.05</td>
<td>0.317</td>
</tr>
<tr>
<td>6</td>
<td>5.38</td>
<td>33.8</td>
<td>0.375</td>
</tr>
</tbody>
</table>

Table 5.3: Damping ratios calculated from decay tests in air

Using the Rayleigh or proportional damping model, the damping matrix can be calculated as a linear combination of the mass and stiffness matrices. See section 3.4 in chapter 3 for more details. The coefficients \( \alpha \) and \( \beta \) in equation 5.14 can be computed using two of the experimental modal damping ratios and the natural frequency of the corresponding modes. The theory in section 3.4 shows

\[
\xi_i = \frac{\alpha + \beta \omega_i^2}{2\omega_i}
\]

(5.14)

where \( \omega \) is the frequency in rad. Using the values \( \xi_2, \omega_2, \xi_6 \) and \( \omega_6 \) from table 5.3 into equation 5.14, a system of two equations with two unknowns can be constructed, hence the coefficients \( \alpha \) and \( \beta \) can be obtained. Equation 3.69 can be rewritten with the computed coefficients, obtaining the damping matrix for the riser model.

\[
C = 0.0555M + 0.0002K
\]
The damping coefficients generate a damping matrix which contains very small values, and the conclusion is that using a proportional damping model based on the experimental decay tests, the damping is not relevant. Figure 5.9 shows a plot of equation 5.14 with the computed coefficients, depicting the damping ratio as a function of frequency.

5.4 Results

All the data analyzed was taken from sections of runs of at least a duration of 60 seconds, to ensure a long enough steady part of each test. The towing carriage needed several seconds from its start position until the steady part of the run was achieved. Depending on the test velocity and tension, this initial transient part of the run, was longer or shorter. For the analysis presented here, each test was examined individually in order to find the steadiest time window in which the data analysis is based. In most of the cases, the time window consisted of the last 40 or 50 seconds of the run, in some
others, because of the modulation of the signals, it was the last 20 or 30 seconds.

The data analysis was first done using sinusoidal mode shapes, and secondly using FEM mode shapes. The differences in results between using the two types of mode shape, were found to be very small. Stated in section 5.2, only the low tension cases showed small differences.

Non-dimensional parameters have been used for the analysis of the VIV response. All the curvatures and displacement amplitudes appear non-dimensionalised with the external diameter of the riser. The reduced velocity has been calculated using the fundamental natural frequency in still water \( (f_1) \) or the dominant frequency cross-flow \( (f_{dy}) \) and in-line \( (f_{dx}) \).

\[
V_1 = \frac{V}{f_1 D} \tag{5.15}
\]

\[
V_{dy,x} = \frac{V}{f_{dy,x} D} \tag{5.16}
\]

The fundamental natural frequency \( (f_1) \) used in 5.15 derives from the eigenvalue FEM calculations described in section 3.3 using as top tension, the mean top tension during the steady part of the test \( T_I \). The dominant frequencies have been obtained finding the peak in the spectra of the dominant modal amplitudes.

If a generic signal \( s \) with \( S \) samples in time, \( s = [s_1, s_2, ..., s_i, ..., s_S] \) is considered, the root mean square value or \( RMS \) \( (\bar{s}) \), the standard deviations \( (\sigma_s) \), the maximum \( (\hat{s}) \), the minimum value \( (\hat{s}) \) and the mean \( (\bar{s}) \) can be calculated as in equations 5.17 to 5.19.

Once local curvatures and displacements were obtained, those statistical values were computed for all the different signals acquired, producing a set of representative values for each run.

\[
\bar{s} = \frac{1}{S} \sum_{i=1}^{S} s_i(t) \tag{5.17}
\]
Curvature measurements and hence displacements are available not only in time but also in space, at several points along the axis of the riser. The standard deviation and the $RMS$ in these cases were calculated first spatially and then temporally. $\sigma_y$ and $\tilde{y}$ have exactly the same mathematical expression, because a cross-flow mean deflection does not exist, therefore, from now its value will be referred as $\sigma_y$. The difference between $\sigma_x$ and $\tilde{x}$ is that in the first, the mean deflected shape is not taken into account. The same applies to the curvatures with $\sigma_{cy}$, $\sigma_{cx}$ and $\tilde{c}_x$. They have been computed as shown below:

$$\sigma_x = \sqrt{\frac{1}{S} \sum_{i=1}^{S} \left[ \frac{1}{N} \sum_{j=1}^{N} \left[ u_{ji}(z, t) - \bar{u}_i(z) \right]^2 \right]}$$ (5.20)

$$\tilde{x} = \sqrt{\frac{1}{S} \sum_{i=1}^{S} \left[ \frac{1}{N} \sum_{j=1}^{N} u_{ji}^2(z, t) \right]}$$ (5.21)

$$\sigma_y = \tilde{y} = \sqrt{\frac{1}{S} \sum_{i=1}^{S} \left[ \frac{1}{N} \sum_{j=1}^{N} v_{ji}^2(z, t) \right]}$$ (5.22)

where $S$ is the number of time samples in the selected time window for the analysis, $N$ is the number of nodes in which the riser has been discretised.

The table below shows the main representative values computed for a test carried out on May 20th (B008) as an example. It corresponds to one of the tests in series D (c) in table 5.5. $\hat{y}$ and $\hat{x}$ are the maxima of the peak displacements at each instant sampled inside the chosen steady time window. $\left( \hat{y} - \bar{y} \right)$ is the same, but with the mean in-line...
deflection subtracted. Following it, several figures are presented, showing aspects of this test run. All vertical axes in the following figures represent the elevation of the riser starting at its bottom, in m. Horizontal axes are non-dimensionalised respect to the model external diameter ($D$). Videos were also generated producing animations of the tests where one can see the response of the riser during the run.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>0.6 m/s</td>
<td>$C_d$</td>
<td>2.35</td>
</tr>
<tr>
<td>$Re$</td>
<td>14850</td>
<td>$\frac{\bar{y}}{\bar{x}} D$</td>
<td>0.82/3.56</td>
</tr>
<tr>
<td>$V_1$</td>
<td>22.8</td>
<td>$\frac{(x-x_0)}{D}$</td>
<td>0.36</td>
</tr>
<tr>
<td>$V_{d,y} V_{d,x}$</td>
<td>6.14/3.07</td>
<td>$z_y z_x$</td>
<td>4.82/4.92</td>
</tr>
<tr>
<td>$T_{t,0}</td>
<td>T_{t,1}$</td>
<td>1529/1610</td>
<td>N</td>
</tr>
<tr>
<td>$T_{b,0}</td>
<td>T_{b,1}$</td>
<td>1339/1420</td>
<td>N</td>
</tr>
<tr>
<td>$n_{d,y}</td>
<td>n_{d,x}$</td>
<td>4/7</td>
<td>$\tilde{c}_y D</td>
</tr>
<tr>
<td>$f_{d,y}</td>
<td>f_{d,x}$</td>
<td>3.52/7.04 Hz</td>
<td>$z_{c_y}</td>
</tr>
</tbody>
</table>

Table 5.4: Data analysis main parameters - example May20B008

Figure 5.10 shows the spatio-temporal variation of the in-line and cross-flow displacement amplitudes for this example. 5.10(a) shows the mean deflected shape over the steady part of the run, 5.10(b) shows the RMS of the in-line deflection, 5.10(d) the RMS of cross-flow deflection and finally, in 5.10(c) and 5.10(e) the instantaneous in-line and cross-flow displacements, can be seen as colour maps. The inclined patterns appearing specially in the response colour maps, indicate the existence of travelling waves.

Another way of showing the response is displayed in figure 5.11, where the in-line mean deflection 5.11(a) and the instantaneous in-line 5.11(b) and cross-flow 5.11(c) deflections are plotted. The central plot shows the instantaneous in-line deflections of the riser model around the static deflection. Solid lines in the last two plots indicate the shape of the riser every three quarters of a second and dashed lines indicate the envelopes during the steady time window, which in this case goes from the tenth second
Figure 5.10: Example: Spatio-temporal plot of non-dimensional response - May20 B008. Vertical axes show elevation along the axis of the riser model in m. Colourmaps show the time series of the non-dimensional motions during 3 seconds.
Curvatures are shown in figure 5.12 in the same fashion as in figure 5.10. Mean curvature appears in 5.12(a), RMS of in-line and cross-flow in 5.12(b) and 5.12(d) respectively. The colour maps in 5.12(c) and 5.12(e) show instantaneous curvatures within the 40th and the 42th second.

Two dimensional trajectories plotted in Figure 5.13 show the overall deformation of the riser in the xy plane at different z positions along the axis of the model. The ten plots on the left show the motion of the riser at different heights whilst the plot on the right shows a 3D view of the 2D trajectories.

The time series of top and bottom tensions and their frequency spectra can be seen in figure 5.14. Spectra of the tensions show most of the time peaks corresponding to the in-line and cross-flow dominant response frequency and its harmonics.
Figure 5.12: Example: Non-dimensional curvatures - May20 B008. Vertical axes show elevation along the axis of the riser model in m. Colourmaps show the time series of the non-dimensional curvatures during 3 seconds.
Figure 5.13: Example: Non-dimensional 2D Trajectories - May20 B008
Figure 5.14: Example: 2D Tensions and spectra - May20 B008
The difference between the tensions can be easily predicted by means of equation 3.5. The increase in both tensions, from the initial values to the mean tension due to the fluid drag, is also predictable by decomposing the total drag force according to the slopes of the riser at each end.

In-line and cross-flow non-dimensional displacement modal amplitudes for the whole experiment are presented in the first column (a) of figures 5.15 and 5.16 respectively. Second column (b) shows only two seconds of the run. The third column (c) shows the spectra of each modal amplitude. It can be seen in this case that a strong multimodal behaviour occurs with several modes running at the same frequency. In-line dominant frequencies are roughly twice those in cross-flow, and in this particular case the frequency peaks are very clear with all the power concentrated there. In this case amplitudes were slightly modulated at different parts of the run, in some others the amplitude modulation was more important.

In some of the test cases, this facts were not as evident as in the example shown, several mode changes were observed. It is believed that they were triggered by external factors such as imperfections in the rails where the carriage was moving. That could have produced the changes in the dominant mode. An example of a test showing the mode change is shown in figures 5.17 and 5.18 taken from run on May 21st (A017).

Peaks in the external accelerometers installed at the top and the bottom of the supporting structure (see Figure 4.6 in chapter 4), indicate these imperfections in the rails when the mode jumps occurred. In other cases there was not an apparent reason. Figure 5.19 shows the time series of the signals acquired during the run with the mode switch (A017), with the external accelerometers. Imperfections in the rails can be appreciated in the acceleration peaks appearing in the time series. Spectra show how the structure was responding at the same frequencies as the riser.
Figure 5.15: Example: Non-dimensional in-line displacement modal amplitudes - May20 B008
Figure 5.16: Example: Non-dimensional cross-flow displacement modal amplitudes - May20 B008
Figure 5.17: Mode switch: Non-dimensional cross-flow displacement modal amplitudes - May21 A017
Figure 5.18: Mode switch: Non-dimensional in-line displacement modal amplitudes - May21 A017
Figure 5.19: External accelerometers and spectras - May20 B008
5.4.1 The Plain Cylinder

This section summarizes data acquired in 149 tests performed with the plain cylinder. In the majority of the cases the sealed tank was filled with water but in some of them, air was surrounding the riser model. The table below shows the letters given to indicate each set of tests, the initial top tension (still water tension) for each set of tests, if the sealed tank was full of water or air, the number of tests of each set and symbols used in all the figures.

<table>
<thead>
<tr>
<th>Series</th>
<th>Initial top tension $T_{t_0}$ (N)</th>
<th>Air/water</th>
<th>Symbol</th>
<th>Number of runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>410</td>
<td>Water</td>
<td>+</td>
<td>19</td>
</tr>
<tr>
<td>Ae</td>
<td>410</td>
<td>Air</td>
<td>*</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>810</td>
<td>Water</td>
<td>◊</td>
<td>76</td>
</tr>
<tr>
<td>C</td>
<td>1175</td>
<td>Water</td>
<td>□</td>
<td>11</td>
</tr>
<tr>
<td>D</td>
<td>1538</td>
<td>Water</td>
<td>○</td>
<td>15</td>
</tr>
<tr>
<td>E</td>
<td>1922</td>
<td>Water</td>
<td>△</td>
<td>16</td>
</tr>
<tr>
<td>Ee</td>
<td>1922</td>
<td>Air</td>
<td>▽</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5.5: Plain cylinder sets of tests

Figure 5.20 shows the initial top tension ($T_{t_0}$) and the mean top tension measured over the steady part of the run ($\overline{T_t}$) for each set, against $V_1$. It can be seen how $T_{t_0}$ was the same at the beginning of each run and how $\overline{T_t}$ increases quadratically due to the effect of the drag on the riser.

In Figure 5.21 the fundamental natural frequency ($f_1$) can be seen as a function of $V_1$ for each set of tests. With increasing flow speed, the tension increases with respect to that in still water. The drag is proportional to the square of the flow speed, so is the tension. Because $f_1$ is related to the mean top tension during each run and it varies because of the drag, $f_1$ varies in the same fashion. Cases with same tension but the tank empty of water have different tension variations along the axis of the riser due to different buoyancy and the values of $f_1$ are higher for the same mean tensions.

Displacements can be shown in many different ways, in the present work the standard
Figure 5.20: Plain cylinder: $T_{t_0}$ and $T_t$ versus $V_t$ ♦ 810 N, □ 1175 N, ◊ 1538 N, + 410 N, * 410 N (empty tank), △ 1922 N and ▽ 1922 N (empty tank)
Figure 5.21: Plain cylinder: $f_1$ versus $V_1$
Figure 5.22: Plain cylinder: Mean of maximum in-line deflection versus $V_1$

deviation, as defined in equations 5.20 and 5.22 has been chosen as representative value. Maxima are not as representative, but they give very useful information about the response. In figure 5.22, each of the points presented is the mean of the maximum ($\bar{x}$) in-line deflections of the riser during a whole run. For each sample in time inside the selected time window, the maximum displacement along the axis of the riser was found, then the mean value in time of all these maxima was found. Given that the data is scattered, the quadratic tendency due to the effect of the increasing drag with flow speed, producing the mean in-line deflection, is very clear. The line shows the best quadratic fit to the data which has the following expression:

$$\frac{\bar{x}}{D} = 6.1 \cdot 10^{-3} V_1^2$$  \hspace{1cm} (5.23)$$

A similar plot can be constructed, as in figure 5.22, but using the RMS value ($\tilde{x}$) as described in 5.21. The data follows a quadratic fit with the equation:
Figure 5.23: Plain cylinder: Standard deviations ($\overline{x}$) of in-line deflections as defined in 5.20 versus $V_1$

\[ \frac{\overline{x}}{D} = 1.9 \cdot 10^{-3}V_1^2 \quad (5.24) \]

Using the standard deviations as defined in 5.22 and 5.20, figure 5.24 is obtained. Now the effect of the mean in-line deflected shape has been removed and only the dynamic part of the response is represented.

In-line values vary up to 0.15 diameters and cross-flow up to 0.55 diameters. The data appears scattered at first sight, both in-line and cross-flow, but if it is grouped according to the dominant mode in the response, as shown in figure 5.24, a strong lock in behaviour can be appreciated. The points are arranged following inclined lines with different slopes and the amplitude of the dominant modes increases linearly with reduced velocity. Moreover, there are discontinuities or jumps between the different branches. More than ten regions can be identified, each one is represented in the figure.
Figure 5.24: Plain cylinder: $\sigma_x$ and $\sigma_y$ versus $V_1$
Figure 5.25: Plain cylinder: Non-dimensional cross-flow and in-line maximums versus $V_1$

with a solid line which is a linear fit of the points associated with a response in a specific mode. Responding dominant cross-flow/in-line modes are indicated for each linear fit. The lower the reduced velocity, the narrower the region in the reduced velocity axis. Slopes are higher at lower reduced velocities and it can be observed how for example, the $1/3$ lock in region covers values of $\sigma_y$ from 0.1 to 0.45 whilst the $7/12$ covers only an amplitude band in the $\sigma_y$ axis that goes from 0.25 to 0.4. Similar features can also be observed by plotting the maximums as in figure 5.25, values up to 1.6 cross-flow and 0.7 in-line are observed.

Cross-flow response is mainly between 2 and 6 times larger than in-line (figure 5.26). The mean value of the points in that figure is $\sigma_y/\sigma_x = 4.1$. A value around 3, was reported in a recent publication by Trim et al. (2005), after testing a long flexible riser circular cylinder excited by VIV under uniform and sheared flow.
Figure 5.26: Plain cylinder: Cross-flow to in-line response ratio versus $V_1$
Using the reduced velocity based on the frequency of the dominant mode \( V_{d_{y,x}} \), as in equation 5.16, figure 5.27 is generated. The amplitude response collapses around a value of 6 in the cross-flow case and around 3 in-line. The clear separated lock-in branches observed in the last figures are now not appearing but an increase of amplitude with \( V_{d_{y,x}} \) can be observed, independent on the mode governing the response.

It was observed that in the upper part of the riser, not subjected to a current, the displacements had the same magnitude as in the lower part. In the cases in which the riser was surrounded by air in its upper part, it is evident looking to figures 5.22 to 5.27 that the displacements were in general higher, not only in-line but also cross-flow.

The only difference, in terms of fluid loading affecting the riser, between the tank being full or empty of water, was the hydrodynamic damping and added mass components in the upper part.

When designing riser pipes using commercial codes, often the fatigue produced by
the in-line response is neglected. The data presented in figure 5.28 shows how the curvatures due to the in-line motion are very similar in magnitude to those produced by the transverse motion, hence in-line fatigue damage can be as important as cross-flow, and should never be neglected. Responding mode number increases linearly with \( V_1 \) and the curvatures do increase with the square of the mode number. Therefore, in-line and cross-flow curvatures increase quadratically with \( V_1 \), and the data can be fitted with equations:

\[
\sigma_{c_y} D = 3.9 \cdot 10^{-7} V_1^2 \quad (5.25)
\]

\[
\tilde{c}_y D = 3.4 \cdot 10^{-7} V_1^2 \quad (5.26)
\]
Figure 5.29: Plain cylinder: Non-dimensional in-line and cross-flow dominant frequency versus $V_1$
Non-dimensional frequencies, both in-line and cross-flow are presented in figure 5.29. The cross-flow non-dimensional frequency could be considered the same as the Strouhal number if it is assumed that the frequency at which the riser is responding is the same as the vortex shedding frequency. The in-line response is practically always double the cross-flow frequency, as can be seen in the plot. In a few cases, the in-line frequency was three times higher than the cross-flow frequency. This happened at the places where the jumps or discontinuities between the lock in regions are found and indicates that several modes are competing to be the dominant with similar modal amplitudes. The lock-in regions are also very clear in this plot. The higher \( V_1 \) the more concentrated the points are around its mean value. The lower modes respond over a large extent of normalised frequencies. If the dominant frequency is normalised with respect to the fundamental natural frequency, and plotted against \( V_1 \), the same features can be observed. The points in figure 5.30 are arranged in steps corresponding to the fundamental mode and its harmonics. Linear fits are presented with a slope of 0.16 cross-flow and 0.32 in-line. The dominant cross-flow and in-line mode numbers appear plotted against the reduced velocity in figure 5.31.

Equation 5.7 provides a method to decompose the measured signals into time series of the amplitudes of each mode participating in the response of the model. This technique has been successfully applied to data collected in very recent experimental tests by Lie & Kaasen (2006) and in CFD data by Willden & Graham (2004). Trim et al. (2005) used the same technique but combining the signals of strain gauges and accelerometers distributed along the axis of their model. Computing the standard deviation of each displacement modal amplitude (\( \sigma_{A_{x,y_i}} \), with \( i = 1, ..., M \) being \( M \) the maximum observed mode number) using the expression 5.20 and 5.22, one can obtain figures 5.33 and 5.32, where can be seen the evolution of each modal amplitude with \( V_1 \). Response peaks related to the lock-in regions can be observed in these figures. Responses up to the 14\(^{th} \) mode in-line and up to the 8\(^{th} \) were observed. In some cases it is very clear that several modes were driving the response of the riser at the same time, this fact is especially evident by looking at the in-line displacement modal amplitudes in figure 5.33.
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Figure 5.30: Plain cylinder: Normalized $f_{dy}$ vs. $V_{dy}$ and normalised $f_{dx}$ vs. $V_{dx}$
Figure 5.31: Plain cylinder: Dominant cross-flow ($n_{dy}$) and in-line ($n_{dx}$) mode number versus $V_1$
Another observation is that in general, for a dominant in-line mode $n_x$ running at $f_{d_x}$ Hz, it was observed that the in-line mode $2n_x$ showed also response but at a frequency equal to $2f_{d_x}$ Hz. A similar behaviour was observed when analysing the cross-flow data but in the third cross-flow harmonic. When the dominant cross-flow mode $n_y (\approx \frac{1}{2}n_x)$ appeared running at a frequency $f_{d_y} (\approx \frac{1}{2}f_x)$, response was also observed in the $2n_y (\approx n_x)$ cross-flow mode but running at $3f_{d_y} (\approx \frac{3}{2}f_x)$ Hz. These factors were found to be exact when looking to the peaks in the spectra of the modal amplitudes. The in-line plots show two response regions, one at the same $V_1$’s as cross-flow and another one at twice this $V_1$. It can be seen very clearly by paying attention to the displacement modal amplitude of the sixth in-line mode, there is a peak at a $V_1$ of around 40, and another at around 20. The same happens looking to the modal contributions of the seventh mode, there is a peak at a $V_1 \approx 22$ and another one at $V_1 \approx 44$. The two response regions are not so evident in the cross-flow plots, but looking in detail the data, one can observe how the second one is separated by a factor of $\frac{3}{2}$ in the $V_1$ axis. The fourth cross-flow modal amplitude plot depicts a peak at $V_1$ of around 24, and a second one, less important in magnitude, at around 36. The same fact can be seen when observing the fifth modal amplitudes plot, the peaks appear now at $V_1 \approx 32$ and at $V_1 \approx 43$. In Stansby (1976b) and Stansby (1976a) the locking-on behaviour of a rigid circular cylinder and its effects on the base pressure and drag are discussed. The author comments on how the system responds locking-on at the cylinder frequency (primary), half the cylinder frequency (secondary) and a third of the cylinder frequency (tertiary).

The last part of the data analysis was focused on the hydrodynamic forces exerted by the fluid on the riser. The total drag force can be computed by decomposing the signal acquired with the tension load cells installed at the ends of the model (see Figure 5.34) in the in-line direction. One can calculate the angle at the bottom and at the top of the riser using the first derivative of the displacements (slopes) along the axis of the riser. With these angles the tension can be decomposed allowing the calculation of the total in-line force, and hence drag coefficients ($C_d$). In Figure 5.34 a diagram depicts the top end of the riser and how the top tension is decomposed into a component in the direction of the flow. The summation of the projection of the tension on the in-line
Figure 5.32: Plain cylinder: Standard deviation of the time series of the ten first non-dimensional cross-flow displacement modal amplitudes vs. $V_1$. Amplitudes in diameters, from the 1st to the 10th in-line mode.
Figure 5.33: Plain cylinder: Standard deviation of the time series of the non-dimensional in-line displacement modal amplitudes versus $V_1$. Amplitudes in diameters, from the 4th to the 13th in-line mode.
Figure 5.34: Tension decomposition into drag

$x$ axis, at the top ($T_{ts}$) and the bottom ($T_{bx}$), gives the total in-line force on the riser model. Dividing the total in-line force by the free stream dynamic pressure ($\frac{1}{2}\rho V^2$) times the diameter ($D$) and the length subjected to flowing water ($L_s$), as in equation 5.27, one can obtain the drag coefficient.

$$C_d = \frac{T_{ts} + T_{bx}}{\frac{1}{2}\rho V^2 L_s D} \quad (5.27)$$

where

$$T_{ts} = T_t \sin \phi_t = T_t \sin \left[ \arctg \left( \frac{du}{dz} \right) \bigg|_{z=L} \right]$$

$$T_{bx} = T_b \sin \phi_b = T_b \sin \left[ \arctg \left( \frac{du}{dz} \right) \bigg|_{z=0} \right]$$

$C_d$'s are shown in figure 5.35. The points appear again grouped into lock-in regions according to responding dominant modes. As already seen in the case of the amplitudes (see figure 5.24) the $C_d$'s increase linearly with $V_1$ inside each region, and the solid lines in the graph are the linear fit to the points inside each region. Beside each of these lines it is indicated the responding dominant cross-flow/in-line mode. Most of the $C_d$ values are concentrated within the range from 1 to 2.8. These high values contrast with the numerous studies carried out during the past decades on rigid stationary cylinders, were $C_d$ values much lower were reported for the same Reynolds number as in our experiments. In the case of long flexible cylinders, similar values of drag
Figure 5.35: Plain cylinder: $C_d$’s versus $V_1$ and even higher have been found in past experiments (Vandiver 1983). Experiments carried out under similar conditions to the one presented here showed similar values of drag coefficient, between 1.5 and 2.7. The test pipe had an aspect ratio of 767, it was towed horizontally under tension (de Wilde & Huijsmans 2004). The fundamental natural frequencies of the pipe were about 1.5 Hz for tensions of 1 kN and the tests were carried out in the sub-critical regime for Re slightly higher than the ones presented in this thesis. They comment on the drag amplification due to the VIV lock-in process suffered by the pipe, leading to the mentioned $C_d$s.

The dependence of the drag coefficients with the riser cross-flow response appears to be clear in figure 5.36, where the coefficients are plotted against $\sigma_x$ and $\sigma_y$. $C_d$ values of around 1 are only observed in tests in which the riser was responding with very low amplitudes. The drag coefficients observed when the riser model was undergoing large amplitude vibrations, was between two and two and a half times bigger than those cor-
responding to case where the cylinder was practically static. Vandiver (1983) reported the same phenomena in his experiments at Castine, Maine. Figure 5.36 shows also very clearly the dependence of the in-line motions with $C_d$. Second order polynomial fits are presented in the graphs and described by equations:

$$\frac{C_d}{C_{d_0}} = 1 + 3.6 \frac{\sigma_y}{D} - 1.9 \left( \frac{\sigma_y}{D} \right)^2$$

(5.28)

$$\frac{C_d}{C_{d_0}} = 1 + 14.7 \frac{\sigma_x}{D} - 40.8 \left( \frac{\sigma_x}{D} \right)^2$$

(5.29)

where $C_{d_0}$ is the drag coefficient of a stationary rigid cylinder at the same Reynolds number as in the experiments. The Re regime in our experiments is subcritical with values between approximately 2800 and 28000. In this range the $C_d$ for a stationary rigid cylinder is quite stable, and a value of $C_{d_0} = 1.1$ (Sumer & Fredsøe 1997) can be applied to our Reynolds number range.

Vandiver (1983) carried out experiments on a long flexible cylinder model of approximately 22.9 m in length with an aspect ratio of approximately 750. Reynolds numbers up to 22000 were achieved, with uniform tensions along the length because the model was disposed horizontally. He proposed a modification (eq. 5.31) of an equation (5.30) first presented by Skop et al. (1977), to fit his experimental data. Evangelinos et al. (2000) performed DNS calculations of the flow coupled to a structural model that allowed them to find not only the mean forces on the cylinder, but also the fluid force distributions. In their calculations, the response was dominated by the second mode in one case with standing wave behaviour and in another with traveling waves. In their paper, they comment on the need for benchmark experiments to compare their DNS data. They proposed equation (5.32) as a fit to their data.

$$\frac{C_d}{C_{d_0}} = 1 + 1.16 \left[ 1 + \left( \frac{2\gamma}{D} \right) \left( \frac{f_n}{f_s} \right) - 1 \right]^{0.65}$$

(5.30)
Figure 5.36: Plain cylinder: $C_d$’s versus $\sigma_x$ and $\sigma_y$
In equation 5.30, $\tilde{y}$ is twice maximum cross-flow amplitude or the peak-to-peak amplitude (if a standing wave response is assumed), $f_n$ is the dominant frequency and $f_s$ is the vortex shedding frequency. $\tilde{y}$ in equations 5.31 is the RMS of the peak-to-peak amplitude. $C_{d_1} = 1.4$ in equation 5.32 is the drag coefficient at the nodes as reported in Evangelinos et al. (2000).

Vandiver’s equation provides a better fit to our data, as it can be seen in figure 5.37.

The following chapter of this thesis is devoted to develop an alternative way of com-
puting the $C_d$ and the hydrodynamic force distribution along the axis of our cylinder model. This method is completely independent to the one used in this chapter which is based on load cell measurements. The computation of the mean force distribution along the axis of the riser for each one of the tests and its integration in space provides a method to find the total force acting and hence the mean drag and lift coefficients, see chapter 6.

In the past decades considerably effort has gone into trying to find the maximum amplitude attainable for a structure undergoing VIV. Maximum amplitudes are presented as a function of a parameter that combines mass and damping characteristics of the structure. When the value of this parameter increases, the response of the structure decreases. Griffin (1982) proposed the following expression for this reduced damping, stability or mass-damping parameter:

$$S_G = \frac{8\pi^2 S_t^2 m \xi}{\rho D^2}$$  \hspace{1cm} (5.33)

where the Strouhal number is $S_t = \frac{fD}{2\pi V}$, and the damping ratio is $\xi = \frac{r}{2\omega m}$ being $r$ the constant damping per unit length, $\omega$ the frequency of the vibration in rad/s and $m$ the mass per unit length. Substituting these expressions in 5.33 one obtains another expression independent of the mass (Vandiver 1993).

$$S_G = \frac{r \omega}{\rho D^2}$$  \hspace{1cm} (5.34)

Sarpkaya (1979) suggested an expression to predict the maximum amplitude caused by VIV depending on the combined mass-damping parameter.

$$\hat{y} \frac{D}{D} = \frac{0.32\gamma}{\sqrt{0.6 + \left(\frac{8\pi^2 S_t^2 m \xi}{\rho D^2}\right)^2}}$$  \hspace{1cm} (5.35)

where $\gamma$ is the geometric factor parameter which depends on the type of structural element analyzed. For strings, cables and simply supported beams it has a value of
Figure 5.38: Plain cylinder: Peak to peak amplitude in diameters versus reduced mass-damping parameter ($S_G$)

$\gamma = 1.155$, (Blevins 1984). Griffin (1982) proposed another equation based on a fit to their data, very similar to that of Sarpkaya:

$$\frac{\hat{y}}{D} = \frac{1.29\gamma}{[1 + 0.43\left(\frac{8\pi^2 S^2 m\xi}{\rho D^2}\right)]^{3.35}}$$  \hspace{1cm} (5.36)

Both of these equations are plotted in figure [5.38] together with the data obtained for this thesis. The points on the graph represent the maximum amplitude attained by the riser model, when vibrating at the various combinations of modes observed.
5.4.2 Comparison plain and bumpy cylinder

The amount of data gathered with the bumpy cylinder model is considerably less than in the previous section with the plain model. A total of 38 tests were carried out with the bumpy cylinder. It was tested only during one day in contrast to almost 20 with the plain one. Bumps, as described in chapter 4, were installed in the lower half of the riser exposed to the uniform current. Table 5.6 summarizes the experiments carried out with the bumpy cylinder model.

<table>
<thead>
<tr>
<th>Series</th>
<th>Initial top tension $T_{in}$ (N)</th>
<th>Air/water</th>
<th>Symbol</th>
<th>Number of runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1924</td>
<td>▽</td>
<td>Air</td>
<td>11</td>
</tr>
<tr>
<td>G</td>
<td>1876</td>
<td>*</td>
<td>Water</td>
<td>14</td>
</tr>
<tr>
<td>H</td>
<td>396</td>
<td>+</td>
<td>Water</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 5.6: Bumpy cylinder sets of tests

In general bumps did not provide the suppression expected and observed in rigid cylinder experiments performed by Owen (2001) and Brankovic (2004). In their experiments the cylinder was restricted to move transverse to the flow and under this condition a very good performance of the bumps as a suppression device was observed. They not only minimized the response but also reduced the drag.

In terms of amplitude response and drag coefficients the tests carried out with the bumps show very similar values to the ones presented in figures 5.23 to 5.27 and in 5.35 to 5.36. Changes in the dominant frequencies can be seen in figures 5.39 and 5.40.

The linear fits result in slopes of 0.13 cross-flow and 0.26 in-line, in contrast with the previous values found for the plain model, of 0.16 and 0.32. The excited mode is normally lower when the cylinder is equipped with the bumps, as shown in figure 5.41. Differences appear as well in the FEM calculated natural frequencies. Due to the change in linear mass and buoyancy of the riser $f_1$’s are lower, see figure 5.42.

As a consequence of these frequency changes, the modal amplitude response peaks are
CHAPTER 5. Experimental Results

Figure 5.39: Plain and bumpy cylinders: Non-dimensional frequencies
Figure 5.40: Plain and bumpy cylinders: Non-dimensional dominant frequencies
Figure 5.41: Plain and bumpy cylinders: Dominant modes
Figure 5.42: Plain and bumpy cylinders: Fundamental natural frequencies
shifted to higher values in the reduced velocity axis, see figures 5.43 and 5.44.

It is very difficult to compare long flexible cylinder data against rigid cylinders data, and it is even harder if suppression devices are used. Cases with more aspects in common are probably those in which a long flexible cylinder is vibrating at its first or fundamental mode. Then, at the point of maximum displacement (the mid point if the mode shapes are considered sinusoids) similarities with rigid cylinder response should be observed. The fact that the bumps proved successful in partially suppressing VIV in rigid cylinders, but not in these experiments, even in the fundamental mode response, suggests that the mechanisms by which vortex shedding produces the excitation, are different. It is believed that a key difference is the fact that in the present work the motion was not restricted to cross-flow, and in the previous studies with the suppression...
Figure 5.44: Plain and bumpy cylinders: Standard deviation of the time series of the non-dimensional in-line displacement modal amplitudes versus $V_1$. Amplitudes in diameters, from the 4th to the 13th in-line mode.
bumps by Owen (2001) and Brankovic (2004), the motion was only allowed transverse to the flow. This fact needs further investigation and experiments with a bumpy rigid cylinder able to move in-line with a frequency twice that in cross-flow are recommended for future researchers.

5.5 Comparison with available prediction methods

In this section results obtained with an empirical based commercial code are presented. A license to use SHEAR7 was provided by BP-Amoco as a part of a collaboration agreement. SHEAR7 is a code developed by Prof. Vandiver and his research team at Massachusetts Institute of Technology during the past two decades, and it is probably, the most widely used VIV prediction code. It is a frequency domain mode superposition method that identifies which modes are most likely to be excited, by locating regions of the riser where the reduced velocity is within a certain range of the ideal lock-in condition. It estimates the VIV cross-flow response on beams or cables with linearly varying tension, under uniform or sheared flows. It uses transverse force models obtained during different sets of experiments. It does not calculate in-line response. More information about the code can be found in Vandiver (2002) and Vandiver (2003).

The results computed with SHEAR7, have been compared against experimental results. The same cases were presented together with results obtained with several other codes in a benchmarking exercise in which several institutions took part. A paper was presented at the 8th International Conference on Flow-Induced Vibration (FIV2004) (Chaplin et al. (2004)). The paper was selected for further publication in a special issue of the Journal of Fluids and Structures (Chaplin et al. (2005)). In this section the results produced for this benchmarking exercise are compared only against the experimental results obtained in the Delta Flume test campaign. 14 representative cases were selected from all the experimental data. They are summarized in table 5.7.

Excited modes and their frequencies, peak modal amplitudes of dominant modes, R.M.S. displacements and maxima of RMS displacement are shown for both SHEAR7
CHAPTER 5. Experimental Results

Table 5.7: Tests considered for the comparison

calculations and the experiments in table [5.8]. Figure 5.45 shows the comparison of the RMS transverse displacements for the 14 cases produced. The dashed lines are the measured cross-flow RMS motions and the solid lines are the ones obtained with Shear7.

Table 5.8: Comparison table SHEAR7 - Experiments

SHEAR7 overestimates in every case the maximum of the RMS cross-flow response. It also overestimates the maximum modal amplitude of the dominant mode. It is accurate when predicting the dominant mode and its associated frequency in single mode cases. In cases with a strong multi-modal response, it failed to predict the right modes and their frequencies. In the experiments it has been observed how the frequency of the
dominant mode is present in all the modes taking part in the response, so several modes run at the same frequency whilst SHEAR7 simply relates each participant mode with its corresponding natural frequency.
Chapter 6

Hydrodynamic Force Distribution

6.1 Introduction

One of the main problems when predicting the response of flexible structures subjected to vortex shedding is that there is not a fully reliable model for the fluid loading on a responding cylinder. There is no experimental data in the open literature, about the force distribution along the axis of a riser undergoing VIV. Some efforts have been made by researchers in order to estimate mean drag coefficients and RMS lift coefficients in the past, and these measurements are the basis of empirical models used to predict the response of risers. Researchers working on CFD tools are also uncertain about the forces obtained by coupling CFD to structural models when the riser is undergoing VIV. The methodology described here uses measured response data and a FEM code to calculate the forces due to vortex shedding, acting on the flexible cylinder. The FEM model described in chapter 3 is used in conjunction with the experimental displacement data presented in chapter 4 to find out the hydrodynamic loading on the riser model. First the lift and drag distributions along the axis of the riser model are presented by means of several examples. The last part of the chapter shows the mean $C_d$'s and the RMS of the $C_l$'s. The drag coefficients obtained by inputting the displacements into the FEM model are then compared with those measured with load cells. The reader must
note that the two methodologies to obtain mean $C_d$’s, are completely independent, the displacements were acquired measuring bending with strain gauges and the total forces were obtained using tension load cells at the top and at the bottom of the riser. Researchers working on CFD codes to simulate VIV of flexible risers will be able to use these data to compare with their computations.

6.2 Axial displacements

Equation 3.59 describes the dynamics of the cylinder. Mass and stiffness matrices are calculated by applying the FEM, and they can be considered constant in time and based on the initial or static configuration. Because it is a problem of small displacements, geometric non-linearities can be neglected (see chapter 3), and the terms $\partial u/\partial z$ and $\partial v/\partial z$ are very close to zero and disappear from the strain relationships making the transverse equations uncoupled. Under these circumstances, the mass and stiffness matrices remain constant in time and they can be based on the static configuration. By inputting into equation 3.59, $r$ and $\ddot{r}$, $F$ can be calculated. $r$ and $\ddot{r}$ are matrices formed of column vectors composed of the in-line, cross-flow displacements and rotations and the axial displacements at each node, for each time step. Cross-flow and in-line responses were measured with strain gauges as described in chapter 4. The axial displacements were not measured and they have been calculated by assuming the riser to be inextensional.

If $s$ is defined as the coordinate that gives the length along the axis of the riser, then the length of an infinitesimal part of it can be expressed as,

$$ds = \sqrt{dx^2 + dy^2 + dz^2}$$

dividing by $ds$ and rearranging, it leads to,

$$z_i(t) = \int_o^{s_i} \sqrt{1 - \left(\frac{dx}{ds}\right)^2 - \left(\frac{dy}{ds}\right)^2} \, dz; \quad s_i = 1, 2, ..., N; \quad s_N = L \quad (6.1)$$
which gives the $z$ coordinate for each node in the riser taking into account the instantaneous deformation. The array of springs installed at the top of the riser allowed us to configure different top tensions assuring that the elastic limit of the material was not exceeded. Using the stiffness of the springs and the measured top tension, the top end position ($z_N$ or $z_L$) can be calculated in an independent manner. Figure 6.1 depicts a scheme for the riser in its initial and deformed states. The riser was attached directly to the supporting structure at the bottom by means of universal joints; at the top, it was attached also with universal joints but to an array of springs hanging from the supporting structure.

The equation below gives the top end position,

$$z_N = L - \left( \frac{\Delta T_t}{K_s} \right) = L - \left( \frac{T_t - T_{t0}}{K_s} \right)$$
where $K_s$, the stiffness of the spring, was either 38.1 kN/m or 26.7 kN/m depending on the test case, because different sets of springs were used. $\Delta T_t$ is the difference between the instantaneous top tension $T_t(t)$ and the initial top tension $T_{t0}$. Figure 6.2 shows a comparison for one of the test cases. The time series of the top end position for the 60 seconds of the run are presented in the upper plot, calculated in the two manners.

Another interesting fact that has been investigated is the how the in-line peak amplitudes vary due to the effect of the spring at the top. The mean of the difference between the initial top end position (coincident with $L$) and the instantaneous top end position, against the point of maximum displacement of the mean in-line deflection, have been plotted in figure 6.3. The figure has been obtained by calculating the instantaneous top position for all the test cases and it shows a square root relationship given by,

$$\hat{x} = \sqrt{\frac{\Delta z_N}{0.178}}$$

(6.2)

It is very important to take into account the movement of the top for the force calcu-
Figure 6.3: Non-dimensional peak in mean deflected shape vs. top end vertical displacement

lations, because small vertical top end movements can result in large changes in the mean in-line deflected displacements and vice versa.

6.3 Application of the FEM to the riser model

With the axial displacements calculated, all the displacements and rotations in the matrices $\mathbf{r}$ and $\ddot{\mathbf{r}}$ are available. We can solve the system 3.59 to find the reduced nodal forces $\mathbf{F}$,

$$\mathbf{K}\mathbf{r} + \mathbf{M}\ddot{\mathbf{r}} = \mathbf{F}_Q + \mathbf{Q} = \mathbf{F}$$

The structural damping is not included because in section 5.3 of chapter 5 it was found to be neglectable. The left hand side of the equation refers only to structural forces (stiffness and inertial force), whilst all the external forces (external fluid forces and the
reactions at the supports) are included in the right hand side of the equation. In order to solve the equation no time integration scheme is necessary, because the solution of the system is known at each time step, and it leads to the time series of the reduced nodal forces, \( \mathbf{F} \). The displacement matrices have 5N (5 degrees of freedom per node) rows, and are of the form,

\[
\mathbf{r} = \begin{bmatrix}
    u_{0t=0} & \ldots & u_{0t=s} \\
    -\frac{\partial u_0}{\partial z} |_{t=0} & \ldots & -\frac{\partial u_0}{\partial z} |_{t=s} \\
    v_{0t=0} & \ldots & v_{0t=s} \\
    -\frac{\partial v_0}{\partial z} |_{t=0} & \ldots & -\frac{\partial v_0}{\partial z} |_{t=s} \\
    w_{0t=0} & \ldots & w_{0t=s} \\
    \vdots & \vdots & \vdots \\
    \vdots & \vdots & \vdots \\
    u_{Lt=0} & \ldots & u_{Lt=s} \\
    -\frac{\partial u_L}{\partial z} |_{t=0} & \ldots & -\frac{\partial u_L}{\partial z} |_{t=s} \\
    v_{Lt=0} & \ldots & v_{Lt=s} \\
    -\frac{\partial v_L}{\partial z} |_{t=0} & \ldots & -\frac{\partial v_L}{\partial z} |_{t=s} \\
    w_{Lt=0} & \ldots & w_{Lt=s}
\end{bmatrix}
\]

\[
\mathbf{\ddot{r}} = \begin{bmatrix}
    \ddot{u}_{0t=0} & \ldots & \ddot{u}_{0t=s} \\
    \frac{d^2 u_0}{dt^2} \left( -\frac{\partial u_0}{\partial z} \right) |_{t=0} & \ldots & \frac{d^2 u_0}{dt^2} \left( -\frac{\partial u_0}{\partial z} \right) |_{t=s} \\
    \ddot{v}_{0t=0} & \ldots & \ddot{v}_{0t=s} \\
    \frac{d^2 v_0}{dt^2} \left( -\frac{\partial v_0}{\partial z} \right) |_{t=0} & \ldots & \frac{d^2 v_0}{dt^2} \left( -\frac{\partial v_0}{\partial z} \right) |_{t=s} \\
    \ddot{w}_{0t=0} & \ldots & \ddot{w}_{0t=s} \\
    \vdots & \vdots & \vdots \\
    \vdots & \vdots & \vdots \\
    \ddot{u}_{Lt=0} & \ldots & \ddot{u}_{Lt=s} \\
    \frac{d^2 u_L}{dt^2} \left( -\frac{\partial u_L}{\partial z} \right) |_{t=0} & \ldots & \frac{d^2 u_L}{dt^2} \left( -\frac{\partial u_L}{\partial z} \right) |_{t=s} \\
    \ddot{v}_{Lt=0} & \ldots & \ddot{v}_{Lt=s} \\
    \frac{d^2 v_L}{dt^2} \left( -\frac{\partial v_L}{\partial z} \right) |_{t=0} & \ldots & \frac{d^2 v_L}{dt^2} \left( -\frac{\partial v_L}{\partial z} \right) |_{t=s} \\
    \ddot{w}_{Lt=0} & \ldots & \ddot{w}_{Lt=s}
\end{bmatrix}
\]

(6.3)

The boundary conditions are implicit in the displacement matrices. The transverse displacements at the top and at the bottom are zero because universal pin joints were used. The axial displacement is only zero at the bottom because the top of the riser was allowed to move vertically with the spring system.

\[
\begin{align*}
    u_0 &= 0 & u_L &= 0 & \forall t \\
    \frac{\partial^2 u_0}{\partial z^2} &= 0 & \frac{\partial^2 u_L}{\partial z^2} &= 0 & \forall t \\
    v_0 &= 0 & v_L &= 0 & \forall t \\
    \frac{\partial^2 v_0}{\partial z^2} &= 0 & \frac{\partial^2 v_L}{\partial z^2} &= 0 & \forall t
\end{align*}
\]
$w_0 = 0 \quad , \quad w_L = 0 \quad \forall t$

These boundary conditions imply that the first, the third, the fifth, the 5N-4 and the 5N-2 rows in matrices $6.3$ and $6.3$ are zero. The same row and column numbers need to be eliminated in $K$ and $M$ to solve the system.

In order to obtain the external nodal forces $f(z, t)$ or in matrix form $\mathbf{f}(t)$, it is necessary to consider the integral equations $3.29$ and $3.43$. The external nodal forces $\mathbf{f}(t)$ need to be computed from the reduced nodal forces $\mathbf{F}$. The external loading can be expressed as a linear combination of the external nodal forces and the shape functions, as was done in equations $3.11$ to $3.13$ of chapter $3$ with the nodal displacements.

$$f_x(z, t) \simeq f_x(t) \Psi(z) = \sum_{j=1}^{n_e} f_{xj}(t) \psi_j(z) \quad (6.4)$$

$$f_y(z, t) \simeq f_y(t) \Psi(z) = \sum_{j=1}^{n_e} f_{yj}(t) \psi_j(z) \quad (6.5)$$

$$f_z(z, t) \simeq f_z(t) L(z) = \sum_{j=1}^{n_e} f_{zj}(t) \ell_j(z) \quad (6.6)$$

Substituting them into their respective equations $3.29$ and $3.43$, one obtains,

$$F^e_{x_i} = \int_{z_1^e}^{z_2^e} \left( \sum_{j=1}^{n_e} f_{xj}(t) \psi_j(z) \right) \psi_i(z) dz$$

$$F^e_{y_i} = \int_{z_1^e}^{z_2^e} \left( \sum_{j=1}^{n_e} f_{yj}(t) \psi_j(z) \right) \psi_i(z) dz$$

$$F^e_{z_i} = \int_{z_1^e}^{z_2^e} \left( \sum_{j=1}^{n_e} f_{zj}(t) \ell_j(z) \right) \ell_i(z) dz$$

Observing the expressions for the mass matrix in $3.27$ and $3.42$, the previous equations can be rewritten now for the global system including the $x$, $y$ and $z$ directions, as
where its components are the external nodal forces and moments that give the hydrodynamic loading along the axis of the riser.

6.4 Results

In this section several examples are presented of force distributions along the axis of the riser model used in the experiments. 8 cases have been selected, ranging from low mode response to high mode response in order to show the main features of the computed loads. Two cases which are included in this thesis were presented in the 4th Int. Conference on Bluff Bodies Wakes and Vortex-Induced Vibration (BBVIV-4) held in Santorini in summer 2005 and the work was selected for publication in the Journal of Fluids and Structures, (Huera-Huarte et al. 2006). Two more cases will be presented in an ASME meeting in July 2006, (Bearman et al. 2006). In all these published cases the FEM model didn’t include the axial equation of motion and hence didn’t take into account the movement of the top end due to the array of springs of the tensioning system. All the cases presented in this thesis have been obtained by using the FEM model described in chapter 3 therefore the axial equation of motion was included in the analysis. In all the cases in this section, except the 4th (∇), the riser was surrounded by water over its entire length. The case with the riser model surrounded by air in the upper half is used as a validation case, because it is known in advance that the air damping and the added mass forces are very small, so the calculations should show this. The last case, 9(⋆ Bumpy), corresponds to a bumpy cylinder case which is shown to see the effect of the bumps on the force distribution. The table below contains a summary of the calculations presented in this section, with the main parameters of the run.

Figure 6.4 depicts the cross-flow and in-line standard deviations for all the runs, as presented in section 5.4.1 and the red vertical lines show the cases for which forces
calculated appear here. Runs have been chosen from different lock-in branches with different top tensions and flow speeds, covering reduced velocities, based on the fundamental natural frequency, from slightly over 10, to almost 40. Note that case 9 (bumpy cylinder) does not appear in the figure because it only shows plain cylinder cases.

Each case is presented using four figures which contain different sets of plots. One is devoted to the transverse response and forces, another presents the in-line response and forces, and finally the last two plots show the phase between the displacement and the loads at different positions along the length of the riser for a time window during the run.

The transverse response and load figures (6.8(a), 6.10(a), 6.12(a), 6.14(a), 6.16(a), 6.18(a), 6.20(a), 6.22(a) and 6.24(a)) are all arranged exactly in the same way. The upper left corner plots (a), show the instantaneous cross-flow deflections. The (b) and (d) show the RMS of cross-flow and in-line motions respectively. The lift coefficient ($C_l$) distribution along the axis of the riser appears in (c). Solid lines in all the figures represent different instants a quarter of a second apart during a selected time window of the run. The thick blue line in (c) is the RMS of $C_l$. The colour maps show the spatio-temporal evolution of the $C_l$ magnitudes. In (f) one can appreciate the $C_l$ distribution between the 31st and the 40th second of the run, and in (e) the 36th second is expanded.
Figure 6.4: Standard deviations of non-dimensional cross-flow and in-line displacements as appears in figure 5.24 of chapter 5. The vertical red lines indicate the force calculations presented in this chapter.

All the vertical axes in the figures give the elevation from the bottom end of the riser to the top end at 13.12 m.

A second figure is used for each case to show the transverse motions and the forces at different positions along the axis, with their respective spectra (see figures 6.8(b), 6.10(b), 6.12(b), 6.14(b), 6.16(b), 6.18(b), 6.20(b), 6.22(b) and 6.24(b)). Information about the phase angle between transverse motions and forces is shown by plotting the force against the motion in the selected time window. Column (a) in these figures shows the transverse motion, columns (b) shows the spectra of the motion, column (c) depicts the forces with the spectra in column (d), and finally in (e) is shown the force-displacement plot. The forces are in phase with the motions when the maximum values of the force coincide with the maximum values of motion and similarly for the minima. When this happens, the plot collapses over a inclined line with positive slope. A negative slope in
those plots means out of phase whilst a roughly circular plot means a 90 degrees phase difference.

For the in-line response and force figures (6.9(a), 6.11(a), 6.13(a), 6.15(a), 6.17(a), 6.19(a), 6.21(a), 6.23(a) and 6.25(a)), the instantaneous in-line deflections are shown in (a) with the mean deflected shape and without it in (b). The RMS of the in-line (c) and cross-flow (e) motions are shown in the third and fifth plots of the upper row of plots. The distribution of local drag coefficient ($C_d$), appears in the fourth plot (d) together with a thick blue line which shows the mean $C_d$ distribution. The colour map in the lower part of the figure (g), shows the $C_d$ distribution along the axis of the riser between the 31st and 40th second of the run. In the upper right corner (f), the $C_d$ distribution appears expanded for the 36th second of the run.

A fourth plot shows the in-line motions, forces, their respective spectra and the force-displacement plot at different locations along the axis of the riser (see figures 6.9(b), 6.11(b), 6.13(b), 6.15(b), 6.17(b), 6.19(b), 6.21(b), 6.23(b) and 6.25(b)). The figures have the same axes arrangement as those in the cross-flow case.

A general feature for all the cases is the clear relationship between the cross-flow motions and the local $C_d$s. Maxima of local drag appear most times at a points along the span where the cross-flow motions are the largest. The local drag maxima oscillate between almost 3.5 (see case 8 in figure 6.23(a)) and 6 (see case 6 in figure 6.19(a)). At the cross-flow nodes, where practically no response is observed, the $C_d$ minima usually occur, with $C_d$ values similar to those expected in stationary cylinders at the Reynolds number of the experiments.

The trend in most of the cases is that the lift distribution follows the shape imposed by the nodes, either the cross-flow or the in-line ones. In general the largest cross-flow motions are associated with the highest $C_l$ values as expected. At the cross-flow nodes, the lift magnitude is close to zero, and this happens as well in some in-line nodes (see figure 6.20(a) and 6.18(a)). Local $C_l$ peaks have been found with values up to 5. In practically all the cases shown here, the fluctuating lift is larger than the fluctuations in drag. This fact has been confirmed by several authors, as reviewed in (Norberg 2003)
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for stationary cylinders, and it is obvious looking to the cases presented here, that in flexible cylinders it remains valid.

The colour maps indicate the degree of correlation of the forces along the span. By looking to specific instants (vertical lines in the map) it can be seen how much the forces vary along the length. In the upper part, the forces are smaller even though the cross-flow RMS of the response has the same order of magnitude as in the lower part. In case 4, this fact is even more evident, the sealed tank was empty of water because the vacuum pump was stopped, so the water was surrounding the cylinder only in its lower half. Although the motions in the upper part are similar to those in the lower half, the forces are practically zero as expected.

Animations made using the data suggest that in the lower part, where there is a uniform current, and the vortex shedding is exciting the riser model, the lift coefficient is close to being in phase with the motion during certain periods of time. It is very difficult to see it in the plots presented because it depends notoriously in the time window inside the run that is selected and a conclusive common feature regarding the phase angle between motion and forces at certain points cannot be extracted. In the upper part of the riser, without current, it should be expected that force components related to the acceleration and velocity of the structure, i.e. in antiphase or 90 out of phase with the motions, because added mass and fluid damping are the dominant forces. The spectra plots in figures show what was already commented in chapter 5, the dominant modes in-line run at twice the frequency of those in cross-flow.

Measurements of force distributions along the axis of a flexible circular cylinder undergoing multi-modal vortex-induced vibrations are not available in the literature. Evangelinos et al. (2000) have used a three-dimensional DNS code to determine fluid forces acting on a cylinder responding in its second mode: in one case as a standing wave and in the other as a travelling wave. In their paper they present distributions of predicted instantaneous fluid forces and comment on the need for benchmark experiments that can be used to help validate their computational results. Similar high values of force coefficients but for lower Reynolds number and lower mode number responses, are pre-
presented in their paper. Although they only consider responses up to the second mode showing standing wave behaviour, many of the features of their results are similar to the ones presented here.

The noise in the measured signals and the use of the initial configuration geometry to calculate the stiffness, mass and damping matrices, which are not updated each time step, are believed to be the main sources of error. An error estimation in the force distribution calculations is very difficult to carry out because of the nature of the process in which numerical techniques for differentiation and for modeling the structure (FEM) have been used in conjunction with experimental data. An estimation can be made by looking at the case with the upper half length in air, in which the expected hydrodynamic forces should be practically zero because air damping and added mass forces are very small. The force coefficient maxima values found in the upper part in air for these cases, are approximately 10 to 15% of the maximums resulting in the lower part.

Figure [6.5] shows a comparison of the force distribution in the upper half of the riser model between the calculations resulting from the method presented in this chapter and results from the application of the Morison’s equation. In the upper part where there is no current and the riser model oscillates in still fluid, Morison’s equation should give good results when estimating the forces. The FEM calculations take into account the motion of the riser in both directions and the Morison’s equation results presented here, have been computed taking into account only the transverse motion. Morison’s equation gives the total force acting on a body in an oscillatory flow as a summation of three terms.

\[
F = \frac{1}{2} \rho C_d v^2 d + \rho C_m A \frac{dv}{dt} + \rho A \frac{dv}{dt} = \frac{1}{2} \rho C_d v^2 d + \rho C_M A \frac{dv}{dt}
\]  

(6.8)

The first term in the equation is the drag force, the second the hydrodynamic or added mass and the third is the Froude-Krylov term. The last term in the equation is only necessary in oscillatory flows. \( C_M = C_m + 1 \) and it is called the inertia coefficient. In
the case of a cylinder oscillating in still fluid $C_m$, the added mass coefficient is used instead of $C_M$. $C_M$ and $C_d$ can be found plotted in several references such as Sumer & Fredsøe (1997) and Sarpkaya & Isaacson (1981), as a function of Re and as a function of the Keulegan-Carpenter number $KC = \frac{VT}{D}$, where $V$ is the peak velocity per cycle and $T$ the time period of the oscillation. For the case presented in figure 6.8 a $C_m = 1$ and a $C_d = 1.2$ has been used taken from Sarpkaya & Isaacson (1981). The velocity in the equation is the oscillation velocity of the riser model.

Figure 6.5: Comparison of FEM lift coefficients against Morison’s equation results. Vertical axis are the elevation along the riser model from the free surface (around 6m) to the top end. Each solid lines represents one instant in time every quarter of a second.

The figure shows how the values of the force coefficient are similar in magnitude in the upper part of the riser model. The FEM calculated forces are smaller in the zone near the free surface meaning that in that region, where the current changes from a value of zero to the value given by the movement of the carriage, the flow cannot be considered purely oscillatory. In the area near the opened bottom end of the vacuum tank, some flow recirculation existed due to the movement of the carriage and very complex flow is expected, limiting the applicability of Morison’s equation.
6.4.1 Drag coefficients

Mean drag coefficients based on the riser length exposed to the current, for all the test cases, have been computed using the methodology described in chapter 3. They are compared against the $C_d$'s obtained from the data acquired from the load cells, in figure 6.6. The black symbols have been calculated by resolving the measured tension in the in-line direction, as explained in section 5.4.1 and shown previously in figure 5.35. The red symbols are the values obtained from the response and the FEM model. The total drag forces which produced these drag coefficients have been calculated as the spatial integration of the mean in-line force distributions.

$$C_d = \frac{1}{S} \sum_{i=1}^{S} \left( \frac{F_{x_i}}{\frac{1}{2} \rho V^2 L_s} \right)$$  \hspace{1cm} (6.9)

with the total force calculated at each instant in time as follows,

$$F_{x_i} = \int_0^L f_x(z,t) \, dz$$  \hspace{1cm} (6.10)

Solid lines mark the different lock-in regions, and over them the dominant cross-flow/in-line mode is indicated. Lines along the reduced velocity axis, indicate the various lock-in regions. This figure contains the same cases as presented in section 5.4.1, including the drag coefficients for those experimental runs where the upper half of the riser was in air rather than in water. It can be seen how the $C_d$s obtained through the methodology presented here are very close to the values found using the load cell measurement method, therefore, the same features commented in relation to figure 5.35 are valid.

6.4.2 Lift coefficients

The RMS of the transverse force coefficients is presented in the figure below. Each point in the plot is the RMS along the riser model and in time and it has been obtained
from the force distributions, by applying expression 5.18. Clearly the $C_L$ data is more scattered than the $C_d$ data, but if the data is arranged according to the responding dominant modes the lock-in regions with linearly increasing lift coefficient with reduced velocity are clear. The vast majority of points range from values of 0.75 to 2.5.

The values are considerably larger than those expected of stationary circular cylinders.
Figure 6.7: RMS of $C_L$'s
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(a) Cross-flow force distribution

(b) Cross-flow motions, Spectra and force-displacement plot

Figure 6.8: Case 1 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.9: Case 1 - In-line
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Figure 6.10: Case 2 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.11: Case 2 - In-line
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Figure 6.12: Case 3 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.13: Case 3 - In-line
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Figure 6.14: Case 4 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.15: Case 4 - In-line
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Figure 6.16: Case 5 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.17: Case 5 - In-line
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Figure 6.18: Case 6 - Cross-flow
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Figure 6.19: Case 6 - In-line
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Figure 6.20: Case 7 - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.21: Case 7 - In-line
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Figure 6.22: Case 8 - Cross-flow

(a) Cross-flow force distribution

(b) Cross-flow motions, Spectra and force-displacement plot
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.23: Case 8 - In-line
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Figure 6.24: Case 9, bumpy cylinder - Cross-flow
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(a) In-line force distribution

(b) In-line motions, Spectra and force-displacement plot

Figure 6.25: Case 9, bumpy cylinder - In-line
Chapter 7

Main findings and discussion of the results

In the first section of this chapter, the main findings and observations are compiled and reviewed to provide a quick reference for the rest of the thesis. Besides each of the important results the figures and plots where the findings have been inferred from, are pointed out. The second section is devoted to a discussion about the main findings and the output and significance of this research work.

7.1 Main findings

7.1.1 Response

A series of experiments have shown the behaviour of a riser model subjected to a stepped current. The next points summarize the main findings.

– Reduced velocities based on the fundamental natural frequency in still water up to 47 were achieved during the experiments.
The riser model responded in modes up to the 8th cross-flow at a frequency of approximately 6.15 Hz and the 14th in-line at 12.3 Hz (figure 5.31).

The fundamental natural frequencies ($f_1$) in still water were found to be in the range from 0.4 Hz to 1.1 Hz, depending on the applied top tension (figure 5.21). $f_1$ increases quadratically with $V_1$.

The standard deviations of the response reached values of 0.6 diameters cross-flow and 0.17 in-line (figure 5.24).

The in-line and cross-flow responses were clearly organized in lock-in regions according to the different dominant modes when plotted against the reduced velocity based on the fundamental natural frequency in still water. The in-line and cross-flow lock-in regions are clearly related, they are bounded by the same reduced velocity values. Inside each lock-in branch the amplitudes increased linearly with increasing $V_1$, the change between the different lock-in branches is abrupt with amplitudes dropping suddenly about 25% when the dominant mode changes to the next mode (figure 5.24). In the zones were the change in mode happens, there is overlapping between the branches, and at the corresponding reduced velocities the system was found to be unstable, meaning that small external disturbances could produce a change in the dominant mode (figure 5.17 and 5.18).

When the response data is plotted against the reduced velocity based on the dominant frequency of oscillation, all the points collapse in a narrow reduced velocity band that goes from 4.5 to 7 cross-flow and from 2.5 to 3.5 in-line (figure 5.27).

Maxima were observed at values of around 1.4 diameters in the cross-flow direction and almost 0.7 (over the mean) for the in-line deflection (figure 5.25). Slightly higher values of around 1.6 cross-flow were found in cases in which the riser model was not fully surrounded by water. Cases with the tank empty of water showed in general standard deviation and maximums of the amplitudes larger than those surrounded by water all along the length of the model (figures 5.24).
and (5.25). Travelling waves played an important role in defining the maximum amplitudes achieved.

- The ratio between the standard deviation of the cross-flow and in-line response has a mean of 4. Most of the cases show ratios between 2 and 6 (figure 5.26).

- The displacements found in the upper part of the riser (still water) were greater than those observed in the lower half, where the riser model was subjected to the current.

- The mean of the maximum in-line deflections reached values of 14 diameters (figure 5.22), increasing quadratically with $V_1$ as expected, due to the effect of the increasing drag.

- Standard deviations of isolated modes reached values of up to 0.55 diameters cross-flow and up to 0.15 in-line. The multi-mode behaviour was more evident in high mode number response cases. The amplification region in the modal amplitudes is evident in the modal amplitudes plots, where it can be seen the overlap between the different modes to produce the overall response (figures 5.33 and 5.32).

- The cross-flow and in-line (once the mean deflection was removed) mode shapes are close to being sinusoidal. Although FEM estimated mode shapes are a better approximation to the real deflections of the riser at low tensions (in the high tension cases there are small differences) the results to which both options yield are practically identical. The mean deflection showed the maximum displacement value at a point around 3.2 from the bottom of the riser.

- Curvatures of the same order of magnitude were found cross-flow and in-line, indicating that the fatigue damage arising from each plane, could be very similar (figure 5.28).

- Several modes near the dominant were contributing to the response, all running at the same frequency. This happened in both orthogonal planes (figures 5.33 and 5.32).
The in-line motion was characterized by a dominant frequency which was twice the cross-flow one (figure 5.30). In most of the cases harmonics are found but with very low modal amplitude even when considering to curvature or acceleration. When \( n_y \) is the dominant cross-flow mode at a frequency \( f_y \), the \( 2n_y \) mode runs at \( 3f_y \) but with very low modal amplitude. At the same time if \( n_x \) is the dominant in-line mode running at \( f_x \), the \( 2n_x \) appears to run with very low modal amplitude at \( 2f_x \) (figures 5.33 and 5.32).

Non-dimensional frequencies (Strouhal number if the frequency of the dominant mode is considered the same as the vortex shedding during the lock-in) of about 0.16 cross-flow and 0.32 in-line were found (figure 5.29).

7.1.2 Hydrodynamic loads

The mean drag coefficients oscillate in a range that goes from 1 to 2.7. Most of the values are concentrated in the band 1.75 to 2.7 meaning that they are between 50% and 125% higher than those expected for stationary cylinders at the same Reynolds numbers (figure 5.35).

The mean drag coefficients are arranged according to the same lock-in regions as seen in the displacements. The value of the drag increases linearly with \( V_1 \) inside the branch until a point where it drops suddenly to lower values at the same time as the dominant mode switches to the next (figure 5.35).

RMS of lift coefficients exhibit the same lock-in branches when plotted against \( V_1 \). The values oscillate between 0.75 and 3, but most of the points are concentrated in a band that goes from 1 to 2.5 (figure 6.7).

Local drag coefficient maxima at points along the riser model reach about 6 and peaks in local lift coefficient reach values of about 4.5, both correlated to positions where the combined cross-flow and in-line amplitudes are the highest.

Fluctuating lift is larger than the fluctuating drag as happens with the motions.
− The oscillating forces in the part not subjected to a current are clearly smaller than the ones found where the vortex shedding is acting, even when the motions are of the same magnitude.

− The cases responding at high mode numbers present smaller oscillating local drag coefficient than those responding at low mode numbers.

− Drag coefficients are well correlated in the lower part of the riser where there is current. In the transverse case, the lift is related to the mode shape in the part subjected to current and the correlation is higher in the upper part.

− The drag coefficients appear to be in phase with the in-line motion in the lower half part of the riser model, in practically all the cases. For the lift it is not as evident and the phase between motion and force appears to be random.

7.2 Discussion of results

The riser model studied shows a strong lock-in behaviour. This was also observed in previous work by King (1995) in an experiment with a cable with slightly higher mass ratio. This lock-in behaviour means that the vortex shedding process is dominated by the motion in a specific mode. The model responds by locking-in a mode whose natural frequency is close to the frequency dictated by the Strouhal relationship and the same applies in-line at a mode whose natural frequency corresponds to twice the cross-flow dominant one. The cylinder responds vibrating over a range of flow velocities in which the cylinder controls the shedding, synchronizing the multi-mode motion with the shedding of vortices. Under this condition the structure vibrates in almost a standing wave manner combined with traveling waves that appear to be more important in the cases with the tank empty of water where it has less ability to dissipate energy through damping. According to these lock-in branches, and comparing the response with a rigid cylinder free to oscillate transversely to the flow at similar reduced velocities, one can conclude that the riser is never able to achieve other than the initial branch of response. As the amplitude increases linearly inside the lock-in branch, there is a point
where it drops suddenly to a smaller amplitude and the response changes to a higher mode. There are no apparent lower and desynchronization branches at the end of each lock-in branch, as reported in past literature for rigid cylinder sections, Williamson & Govardham (2004). Moreover with increasing reduced velocity the amplitude does not anywhere decrease inside each lock-in region, it keeps growing up until the next mode is excited. The lock-in branches overlap near the change of mode reduced velocity, and the lock-in at these reduced velocities is less stable than at reduced velocities falling in the middle of the branch. Small external disturbances can change the dominant mode to the next one. These facts and the presence in the data of only one important dominant cross-flow frequency and one in-line dominant frequency, related by a factor of 2, suggests that the 2S mode of shedding is the main vortex structure for this type of system.

Although several modes contribute to the response running at the same frequency, there is in all the cases a clear dominant mode that drives the motion of the riser model. The fact that the region where energy is transferred from the flow to the model, drives the overall response (in this case only the lower half is subjected to the current and the vortex shedding process) suggests that in a case with an equivalent uniform flow all along the length of the riser model, the behaviour would be very similar because there would still be only one frequency involved. A natural continuation of this experiment would be to use the same riser model but in a sheared flow profile. That would allow us to determine how the modes compete to be dominant in a system where different shedding frequencies are present. The competition process between the different energy input regions in sheared profiles is still not well understood and more work needs to be done. One thing is clear, in order to predict the response of a system subjected to a sheared flow, finding the dominant mode and how it affects or disrupts the shedding in the other regions and vice versa, as well as how the imputed energy is divided into the other participant modes, are crucial points. Vikestad (1998) in his doctoral research studied how response of a cylinder subjected to uniform flow was affected by the introduction of motions in the supports, at different frequencies to that of the vortex shedding. One of the main conclusions was that depending on the
amplitude of the support motions the cylinder response was attenuated to a certain degree, but the process was still dominated by the fluid-structure interaction and not by the external frequency introduced through the supports.

It is believed that the observed mode overlapping, and the consistent change in the dominant mode as the amplitude increases inside each lock-in range, has to do with the fact that the added mass is not constant inside the lock-in branch Vandiver (1993). As the riser magnifies its response, the added mass distribution varies, producing a variation in the natural frequencies which may become closer to the shedding excitation producing the jump to the next dominant mode.

In practically all the previous studies on the response of rigid cylinders the system was restricted to move only transversely with one degree of freedom. Jauvtis & Williamson (2003) suggests in his work that all the main features found in recent years for cylinders undergoing one degree of freedom motion, are applicable to those systems free to move in-line and transverse to the flow. It is evident from the present work that in a system such as the one studied here, a flexible pin-ended circular cylinder, the in-line motions are as important as the cross-flow ones to understand the overall response due to the fluid-structure interaction, and only some of the characteristic features of 2D sectional studies are applicable.

The suppression bumps, tested in the riser model in some of the runs, have not been effective as expected in reducing the amplitudes or the drag of the system. In previous studies by Owen (2001) and Brankovic (2004) on rigid circular cylinders restricted to move transversely, the bumps reduced notably the amplitudes and the drag coefficients. The shape of the response plot was similar to that of a plain cylinder and the phase change at the reduced velocity associated with the maximum amplitudes was still happening. One of the main differences between the physics of the suppression mechanism for bumpy and straked cylinders was indeed, the change in phase (displacement-lift force) which was totally avoided with the strakes but not by the bumps. Trim et al. (2005) showed good VIV response suppression capabilities of the strakes on a long flexible cylinder with strake coverages of more than 80%, so the strakes carry on the
suppression characteristics from 2D sectional studies to flexible cylinders. The increase in drag generated by the strakes (Brankovic 2004), and the associated increase in tension, could be in part reducing the response, because at a specific flow speed and dominant mode, an increase of tension supposes a decrease of reduced velocity inside the actual lock-in branch, associated with a reduction of the amplitudes. Bumps do not increase the $C_d$ for flexible cylinders, they practically are not modified and it was seen before that they were able to reduce the drag for fixed rigid sections. The continuously variation of the orientation and inclination to the flow according to the instantaneous shape and the 2D trajectories when the cylinder is undergoing the vibrations, together with the freedom to move in-line and transversely to the flow, could be the main causes to explain the lack of effectiveness of the bumps.

The drag coefficients reported in this thesis are higher than those reported in past literature referred to rigid cylinders, so there is an obvious drag amplification related to the cylinder multi-mode motion. They are found to be in agreement with similar experiments carried out with long flexible cylinders (Vandiver (1983) and de Wilde & Huijsmans (2004)). The $C_d$ increases linearly with the reduced velocity inside each lock-in region amplified by the increasing motions. $C_l$ shows this amplification too, inside each lock-in region related to the linearly increasing value of the motions. Vikestad (1998) and Brankovic (2004) in their theses present peak values of total lift and drag coefficients over 3 for a towed rigid cylinder, at the reduced velocities at which the cross-flow motion was the largest. Even though we don’t have experimental evidence, the fact that not only the cross-flow but also the in-line motion increases with the reduced velocity in each lock-in branch suggests that the increase in the force could be related to an increase in the coherence of the shedding along span sections of the riser model. The vortex structures would shed from the cylinder in spanwise cells, in a more organized fashion as the response increases through the lock-in branch, making the loads more coherent, until the next mode is excited. The amplification of the forces is not only related to the cross-flow motion but also to the in-line motions.

Regarding the presently available prediction methods, they have improved in last few years and they provide solutions that are closer to the measurements. Several measure-
ment cases were compared against results produced with one of the most extensively used empirical prediction codes, SHEAR7 (Vandiver 2003). Results are very promising, the program predicts the dominant modes and their frequencies very accurately, but in general it fails by overestimating the cross-flow motion as well as the modal amplitudes. An exercise in which these results were also presented, showed blind comparisons between the measurements and several prediction codes, (Chaplin et al. 2005).

In general all the CFD codes underpredicted the cross-flow motions by 10% to 30% and the mean in-line deflections (associated with the drag coefficient) by 20% to 40%. The RMS values of the computed curvatures were between the 20% and the 180% of the measured ones. The general feeling is that even though it is observed that there has been an improvement in the predictions with respect to past work (Larsen & Halse 1995), there is not a completely reliable tool to predict the VIV behaviour of flexible pipes and that the CFD codes do not produce better results than those based on experimental data. A clear advantage of empirical tools is that they are very fast and easy to use, whilst numerical codes require normally large computer resources and in general are complex to use. The achievable Reynolds numbers are also a limiting factor when attempting 3D DNS calculations, but the results shown by several authors (Evangelinos et al. 2000) with these methods are very promising.
Chapter 8

Conclusions and recommended future work

8.1 Conclusions

The multi-mode response of a flexible circular cylinder has been studied with experimental techniques by using a riser model with a realistic mass ratio ($m^* = 3$) and with an aspect ratio ($L/D = 470$). The Reynolds numbers ($2800 < Re < 28000$) were inside the sub-critical regime and even though they are an order of magnitude away from realistic offshore regimes ($Re > 10^5$), the experiments have given us a good insight and an improved understanding of the VIV physics.

Strain gauges have proved to be successful in obtaining curvature and displacement measurements for a system responding at high mode numbers, together with the data analysis and modal decomposition techniques presented in the thesis. It is shown how the in-line and cross-flow curvatures are of the same order of magnitude therefore in-line motion cannot be neglected when designing sub-sea systems. The response is characterized by a succession of strong lock-in states or branches inside of which, the response dominated by a specific mode, increases in amplitude with flow speed. The
results suggests that the riser model inside each lock-in state characterized by a specific dominant mode, behaved in a similar fashion to a rigid cylinder in its initial branch of response, never achieving the lower and desynchronization regions.

The Griffin plot of vibration amplitude versus a mass-damping parameter has been obtained, showing the data obtained from our experiments, compared against expressions proposed in past literature (Sarpkaya 1979) and (Griffin 1982) that match quite well the maximum attained amplitudes in our experiments.

Drag and lift amplification have been observed and it is shown that they can be related to the effect of the increasing response in each lock-in branch. Expressions that relate the $C_d$ with the in-line and cross-flow motion (equations 5.28 and 5.29), have been proposed as well as plots that provide the mean $C_d$ and the RMS of $C_l$ with varying reduced velocity.

The time-dependent transverse and in-line fluid force distribution on a vertical tension riser model undergoing vortex-induced vibrations in a stepped current, has been obtained from measured displacements by using an indirect finite element technique. The methodology can be applied to other sets of well conditioned data to investigate the force distribution along the axis of flexible systems undergoing VIV. This methodology and the resulting data contributes to an improved understanding of hydrodynamic loading produced by vortex-induced vibrations. $C_d$ and $C_l$ peak values of over 4.5 have been shown correlated to the largest combined $xy$ motions along the riser model. Values near the ones expected for stationary circular cylinders at the same Reynolds numbers as in our experiments are found in the $xy$ nodes, where practically there is no displacement in either direction.

New opportunities for validating flow-induced forces computed by CFD coupled with structural codes are also provided by the present work. From the computational point of view it is clear that there is a need to improve the capabilities of actual codes, not only of those based on CFD but also the empirical ones based on experimental data. The CFD codes are of special interest due to their potential, in conjunction with FEM structural codes, to model suppression devices and different geometries. This is
something that in the future is going to become a necessity in order to design more reliable systems.

8.2 Recommended future work

1. Sheared flows:

The lack of quality data regarding flexible cylinders in sheared flow conditions is another problem that restricts the validation and development of numerical and empirical codes. Experiments under similar conditions to the ones presented here but in sheared flow situations, would provide valuable data which in conjunction with the analysis techniques presented in this thesis, would yield new information that would improve our understanding of the VIV physics.

2. Vortex dynamics and wake structure:

Flow visualization and PIV (Particle Image Velocimetry) in the wake of a flexible cylinder undergoing VIV, would illuminate the flow structures being developed in the near wake. It would allow us to understand the spanwise correlation, measure the shedding frequencies to certify that under lock-in it merges with the frequency of the motion and finally it would clarify if the vortex structures are the same as the ones expected (2S) in the initial branch of response of oscillating rigid cylinders. It would also provide interesting information to compare with the force distributions presented in chapter 6 of this thesis, in order to see the relationship between the estimated forces and the flow patterns producing them.

3. Suppression systems:

In the light of unexpected results obtained with the suppression bumps, it would be very interesting to check the effectiveness of the bumps, in an experiment with a rigid cylinder section allowed to move with 2 degrees of freedom, in an arrangement in which the in-line motion would respond with a frequency twice that in cross-flow. As already pointed out during the thesis, previous work done
in the Department of Aeronautics showed how the bumps were successful in reducing the amplitudes of vibration and the drag coefficient when the cylinders were restricted to move transversely to the flow. It is believed that one of the main reasons why the bumps did not work as expected, could be the intrinsic multi-degree of freedom nature of the flexible systems.

For given flow speed, fluid added mass distribution, the tension controls the mode that is going to be excited. An active tension control system might be used as a suppression mechanism. By setting different tensions and making them vary at different frequencies, the dynamic characteristics could be controlled modifying the response of the system. Experiments with cables subjected to uniform flow speed with a tension control system which would allow changes not only to the magnitude of the tension but also the frequency of application of the changes, would give more insight.

Another suppression device that has proved successful in reducing the response of long flexible cylinders (Trim et al. 2005) and rigid cylinders (Brankovic 2004) is helical strakes. It would be very useful to have data to compare the response of the riser model with and without strakes. The degree of coverage needed to suppress the VIV is also a problem that needs more investigation. The industry is very interested in this aspect, because the strakes are the most widely used suppression device and reductions of strake coverage in riser designs, would result in massive reductions of riser installation costs.
Appendix A

Weak Formulation

The need for an integral form of equation (3.4) comes from the fact that if equations 3.11 and 3.12 are substituted in the governing equation 3.4, it is possible that the resultant system would not always have the required number of linearly independent algebraic equations needed to find the coefficients $u_j(t)$, $v_j(t)$, that represent the solution of our system at each node. The same could happen with $w_j(t)$ when substituting eq. 3.13 in the axial equation of motion, eq. 3.9. A way to obtain the correct number of linearly independent equations is to find the integral weak formulation of the governing equation (Reddy 1993). A weight function $\vartheta(z)$ is used for the purpose, multiplying the governing equations and integrating along the element to find the weak form of the differential equation.

Appendix A.1 Transverse equations of motion

In the case of the transverse equations of motion the procedure is valid for both the equations modeling the motion in the $xz$ and in the $yz$ planes. Introducing the weight function $\vartheta(z)$ into equation 3.4.
\[
\int_{z_1}^{z_2} \vartheta(z) \left[ EI \frac{\partial^4 u(z, t)}{\partial z^4} - \frac{\partial}{\partial z} \left( T(z) \frac{\partial u(z, t)}{\partial z} \right) + m \frac{\partial^2 u(z, t)}{\partial t^2} - f(z, t) \right] dz = 0
\]

Integration by parts once in the second order term, and twice in the fourth order one, allows to obtain the weak formulation of the differential equation

\[
\int_{z_1}^{z_2} \left[ T(z) \frac{\partial \vartheta \partial u}{\partial z \partial z} + EI \frac{\partial^2 \vartheta \partial^2 u}{\partial z^2 \partial z^2} + mw \frac{\partial^2 u}{\partial t^2} - wf \right] dz - \left[ Q_1(z_1^e) - Q_2(z_2^e) - Q_1 \left( \frac{\partial \vartheta}{\partial z} \big|_{z_1^e} \right) - Q_2 \left( -\frac{\partial \vartheta}{\partial z} \big|_{z_2^e} \right) \right] = 0 \quad (A.1)
\]

Because it is a fourth order differential equation we have two primary variables \( u(z, t) \) and \( \frac{\partial u}{\partial t} (z, t) \). That means each node has associated with it two degrees of freedom, one for each primary variable. On the other hand, the secondary variables are:

\[
Q_1(t) = \left[ -T(z) \frac{\partial u}{\partial z} \bigg|_{z_1^e} + \frac{\partial}{\partial z} \left( EI \frac{\partial^2 u}{\partial z^2} \right) \right] \bigg|_{z = z_1^e} \quad (A.2)
\]

\[
Q_2(t) = -\left[ -T(z) \frac{\partial u}{\partial z} \bigg|_{z_2^e} + \frac{\partial}{\partial z} \left( EI \frac{\partial^2 u}{\partial z^2} \right) \right] \bigg|_{z = z_2^e} \quad (A.3)
\]

\[
Q_1(t) = EI \frac{\partial^2 u}{\partial z^2} \bigg|_{z = z_1^e} \quad (A.4)
\]

\[
Q_2(t) = EI \frac{\partial^2 u}{\partial z^2} \bigg|_{z = z_2^e} \quad (A.5)
\]

where the first ones represent shear forces and the second ones bending moments, as seen in section 3.1.1.

**Appendix A.2 Axial equation of motion**

The procedure for the case of the axial equation of motion is the same. The weight functions are introduced in equation 3.1.2 multiplying all its terms and then the integration is developed.
\[
\int_{z_1}^{z_2} \vartheta(z) \left[ m \frac{\partial^2 w(z,t)}{\partial t^2} - EA \frac{\partial^2 w(z,t)}{\partial z^2} - f_z(z,t) \right] dz = 0
\]

Integration by parts of the second order term results in,

\[
\int_{z_1}^{z_2} \left[ m \vartheta(z) \frac{\partial^2 w(z,t)}{\partial t^2} - EA \frac{\partial w(z,t)}{\partial z} \frac{\partial \vartheta(z)}{\partial z} - \vartheta(z) f_z(z,t) \right] dz + \vartheta(z_1^e) Q_1^r - \vartheta(z_2^e) Q_2^r = 0 \quad (A.6)
\]

where the secondary variables, representing tensions, are

\[
Q_1^r(t) = EA \frac{\partial w}{\partial z} \bigg|_{z = z_1^e} \quad (A.7)
\]
\[
Q_2^r(t) = EA \frac{\partial w}{\partial z} \bigg|_{z = z_2^e} \quad (A.8)
\]

**Appendix A.3 Shape functions**

The choice of the weight function \( \vartheta(z) \) in the above equations leads to the different Finite Element methods. The Rayleigh-Ritz method consists of using adequate polynomials to interpolate the nodal values and its derivatives at the nodes. In second order equations, such as the axial equation of motion, Lagrange polynomials are used because there is only the need to interpolate the nodal values of the displacements. Hermite polynomials must be used if the problem consists of a fourth order equation, and the interpolation should include not only the nodal values, but also its first derivatives in space or so called rotations.
Appendix A.3.1 Lagrange family of interpolators

The Lagrange family of first order interpolators $\ell_i(z)$ evaluated at each node of the generic finite element $\Omega^e = [z_1^e, z_2^e]$ is

\[
\ell_1(z) = \frac{z - z_1^e}{z_1^e - z_2^e} \\
\ell_2(z) = \frac{z - z_2^e}{z_2^e - z_1^e}
\]  

(A.9)

In order to implement the method in computer it is more convenient to use the generic finite element $\Omega^R(\zeta) = [-1, 1]$ referred to coordinate system local to the finite element instead of using $\Omega^e(z) = [z_1^e, z_2^e]$ referred to the global coordinates of the system. The integrals resulting of the method will be then solved using Gauss-Legendre Quadrature formulae. To accomplish this change of reference, a change of variable is necessary. The change of variable theorem, which allows a generic function $g(z)$ to change its dependent variable from the global domain $z$ to the local domain $\zeta$, states

\[
\int_{z_1^e}^{z_2^e} g(z) dz = \int_{\phi^{-1}(z_1^e)}^{\phi^{-1}(z_2^e)} g(\phi(\zeta)) \phi(\zeta) d\zeta
\]  

(A.10)

where

\[
\phi(\zeta) = \frac{h^e}{2} (\zeta + 1) + z_1^e \\
\phi^{-1}(z) = \frac{2}{h^e} (z - z_1^e) - 1 \\
\phi'(\zeta) = \frac{h^e}{2} \\
\phi''^{-1}(z) = \frac{2}{h^e}
\]  

(A.11)

where the $'$ indicates differentiation respect the dependent variable. Applying this change to (A.9) one obtains
CHAPTER A. Weak Formulation

\[ \ell_1(\zeta) = \frac{1}{2}(1 - \zeta) \quad \ell_1'(\zeta) = -\frac{1}{2} \]

\[ \ell_2(\zeta) = \frac{1}{2}(1 + \zeta) \quad \ell_2'(\zeta) = \frac{1}{2} \] (A.12)

**Appendix A.3.2 Hermite family of interpolators**

The Hermite family is a set of polynomials that allows us to interpolate the values of the dependent variable and its first derivatives at two given points, in this case the nodes of the finite element. Considering two nodes in each finite element, and two degrees of freedom associated to the primary variables at each node, the Hermite interpolators are necessary to develop the finite element method, and evaluated at each node of \( \Omega^e = [z_1^e, z_2^e] \), are

\[
\psi_1^e(z) = 1 - 3 \left( \frac{z - z_1^e}{h^e} \right)^2 + 2 \left( \frac{z - z_1^e}{h^e} \right)^3
\]

\[
\psi_2^e(z) = -(z - z_1^e) \left( 1 - \frac{z - z_1^e}{h^e} \right)^2
\]

\[
\psi_1^c(z) = 3 \left( \frac{z - z_1^e}{h^e} \right)^2 - 2 \left( \frac{z - z_1^e}{h^e} \right)^3
\]

\[
\psi_2^c(z) = -(z - z_1^e) \left[ \left( \frac{z - z_1^e}{h^e} \right)^2 - \frac{z - z_1^e}{h^e} \right]
\] (A.13)

In local coordinate \( \zeta \) for the generic element \( \Omega^R(\zeta) = [-1, 1] \) after applying the change of variable [A.11] one obtains

\[
\psi_1^c(\zeta) = (2 + \zeta) \left( \frac{1 - \zeta}{2} \right)^2
\]

\[
\psi_2^c(\zeta) = (2 - \zeta) \left( \frac{\zeta + 1}{2} \right)^2
\]

\[
\psi_1^c(\zeta) = (1 + \zeta) \left( \frac{1 - \zeta}{2} \right)^2
\]
\[ \psi_2^e(\zeta) = (\zeta - 1) \left( \frac{\zeta + 1}{2} \right)^2 \] \hspace{1cm} (A.14)

Figure [A.1] shows the polynomials, which are valid to interpolate in the \( x \) and \( y \) directions.

Higher order Hermite polynomials for quadratic finite elements (3 nodes per element) were also used, producing practically identical results. It implied the recalculation of the elemental matrices presented in chapter 3.
Bibliography


